Elections with Opinion Polls:
Information Acquisition and Aggregation

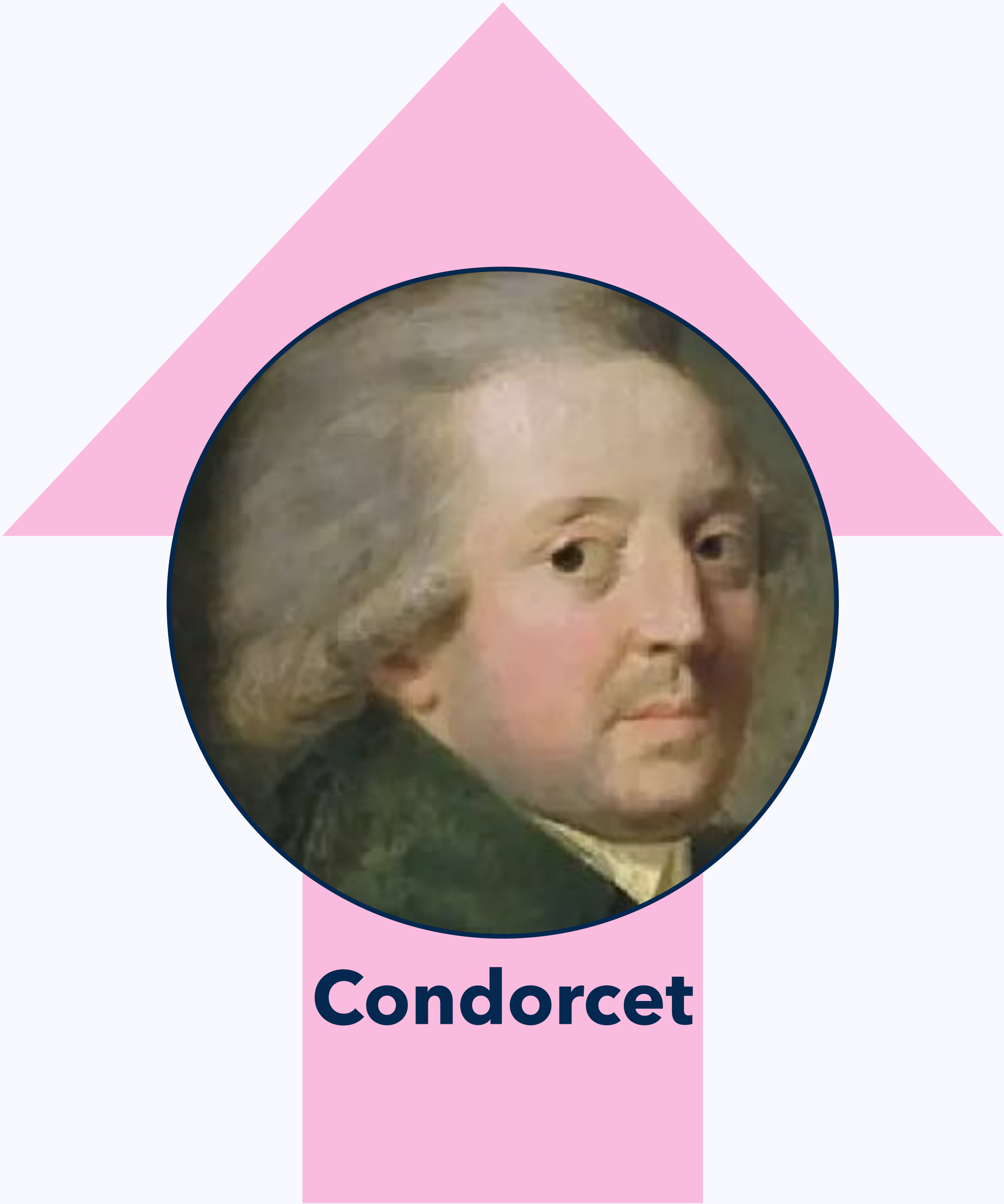
Tetsuya Hoshino ⓡ Andrei Gomberg

CETC 2025

Is Majority Vote “Correct” If Voters Have a Common Interest?

Condorcet

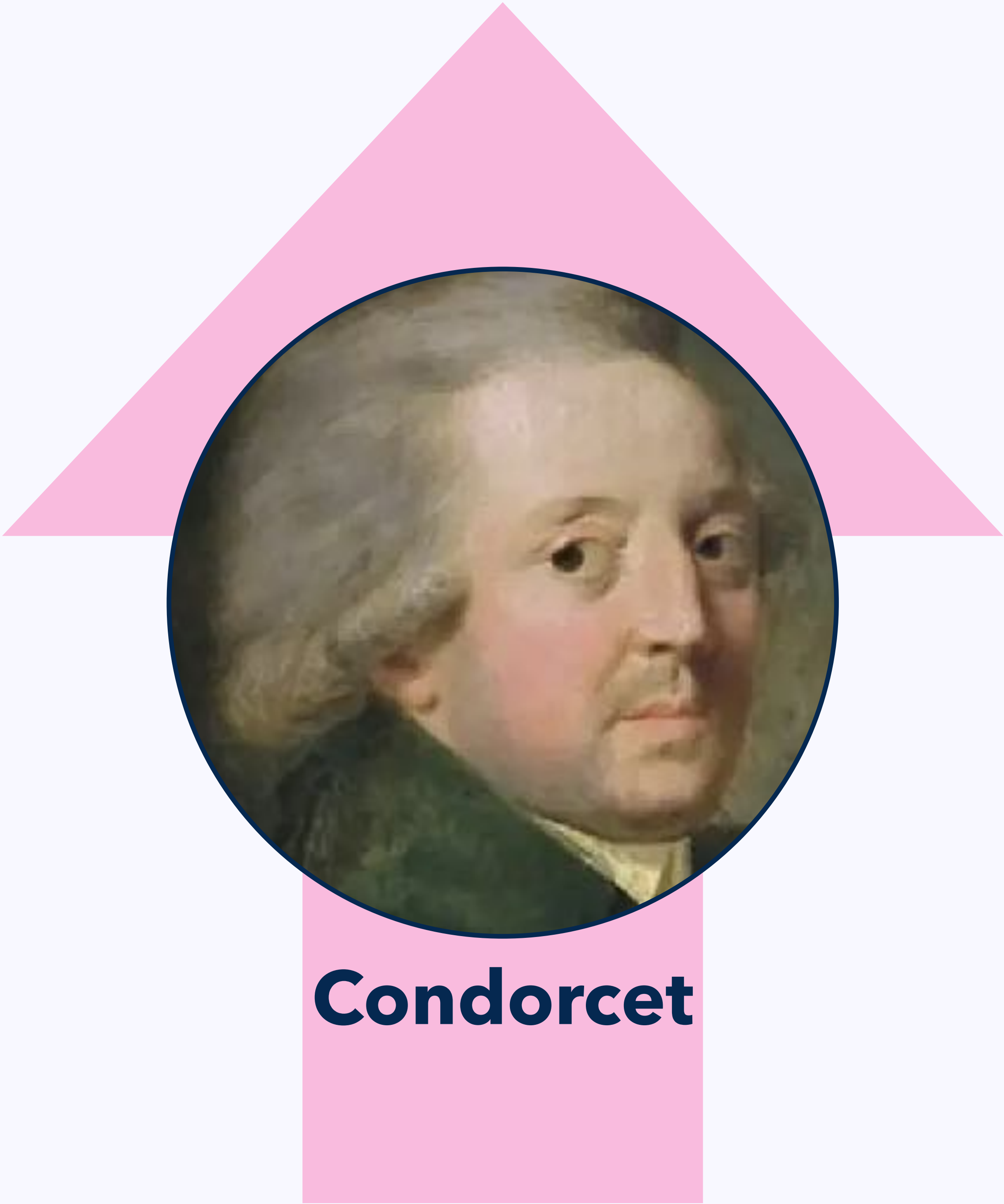
Is Majority Vote "Correct" If Voters Have a Common Interest?

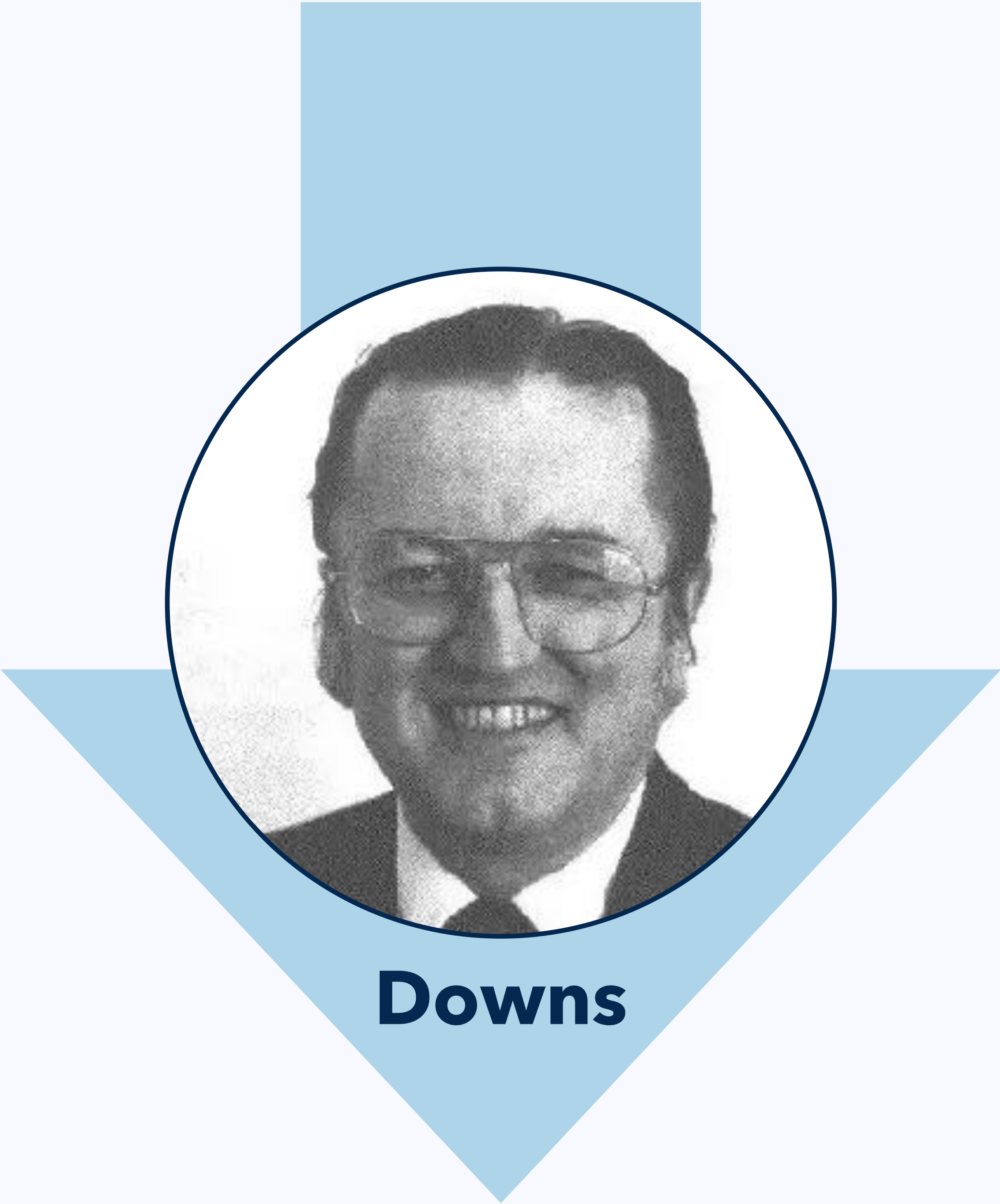


Condorcet's Jury Theorem

- information is exogenous
- #voters↑, Pr(majority is correct) ↑

Is Majority Vote “Correct” If Voters Have a Common Interest?





Condorcet’s Jury Theorem

- information is exogenous
- #voters↑, Pr(majority is correct) ↑

Rational Ignorance Hypothesis

- information is endogenous
- #voters↑, Pr(majority is correct) may ↓

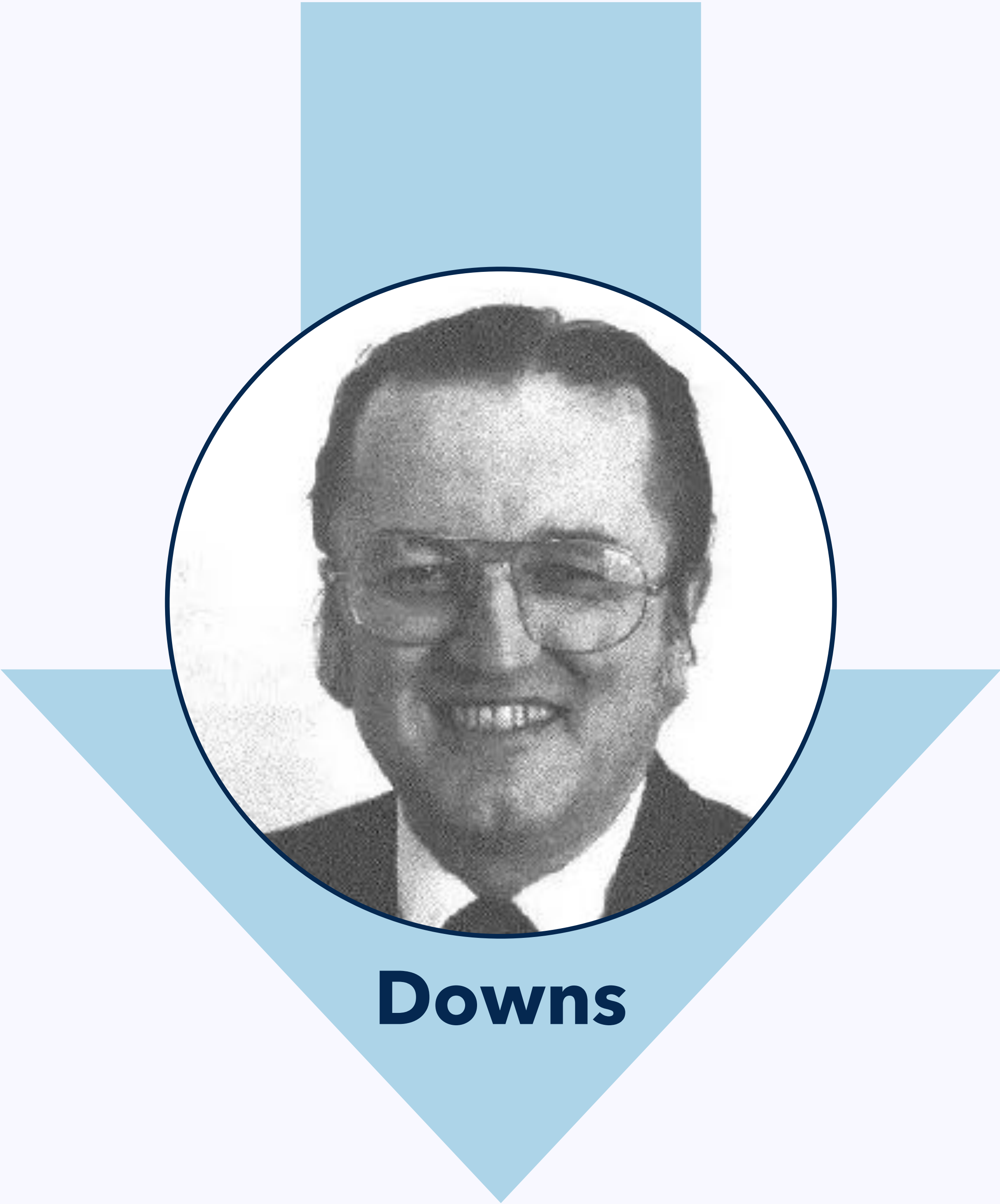
Is Majority Vote "Correct" If Voters Have a Common Interest?

cost of acquiring information

vs

probability of being pivotal

trade-off



Rational Ignorance Hypothesis

- information is endogenous
- #voters↑, Pr(majority is correct) may ↓

Opinion polls allow voters to learn about their likelihood of being pivotal

- if 80 vs 20 $\implies$ Pr(pivotal) $\downarrow$ $\implies$ information $\downarrow$
- if 50 vs 50 $\implies$ Pr(pivotal) $\uparrow$ $\implies$ information $\uparrow$

Opinion polls allow voters to learn about their likelihood of being pivotal

- if 80 vs 20 $\implies$ Pr(pivotal) $\downarrow$ $\implies$ information $\downarrow$
- if 50 vs 50 $\implies$ Pr(pivotal) $\uparrow$ $\implies$ information $\uparrow$

Opinion polls allow voters to learn about their likelihood of being pivotal

- if 80 vs 20 $\implies$ Pr(pivotal) $\downarrow$ $\implies$ information $\downarrow$
- if 50 vs 50 $\implies$ Pr(pivotal) $\uparrow$ $\implies$ information $\uparrow$

Alternatives 1 and 0

- **correct** alternative = state $\theta \sim$ common prior μ

N voters (odd)

- voter i chooses alternative $a_i \in \{1,0\}$
- simple majority rule

$$\textbf{vote share } \bar{a}_N = \frac{1}{N}\sum_{i=1}^{N} a_i \begin{cases} > 1/2 & \iff \text{winner} = 1 \\ < 1/2 & \iff \text{winner} = 0 \end{cases}$$

- common interests

$$u(\bar{a}_N, \theta) = \begin{cases} 1 & \text{if winner} = \theta \\ 0 & \text{if winner} \neq \theta \end{cases}$$

Model

Alternatives 1 and 0

- **correct** alternative = state $\theta \sim$ common prior μ

N voters (odd)

- voter i chooses alternative $a_i \in \{1,0\}$
- simple majority rule

$$\textbf{vote share } \bar{a}_N = \frac{1}{N}\sum_{i=1}^{N} a_i \begin{cases} > 1/2 & \iff \quad \text{winner} = 1 \\ < 1/2 & \iff \quad \text{winner} = 0 \end{cases}$$

- common interests

$$u(\bar{a}_N, \theta) = \begin{cases} 1 & \text{if winner} = \theta \\ 0 & \text{if winner} \neq \theta \end{cases}$$

Model

Alternatives 1 and 0

- **correct** alternative = state $\theta \sim$ common prior μ

N voters (odd)

- voter i chooses alternative $a_i \in \{1,0\}$
- simple majority rule

$$\textbf{vote share } \bar{a}_N = \frac{1}{N}\sum_{i=1}^{N} a_i \begin{cases} > 1/2 & \iff \quad \text{winner} = 1 \\ < 1/2 & \iff \quad \text{winner} = 0 \end{cases}$$

- common interests

$$u(\bar{a}_N, \theta) = \begin{cases} 1 & \text{if winner} = \theta \\ 0 & \text{if winner} \neq \theta \end{cases}$$

Alternatives 1 and 0

- **correct** alternative = state $\theta \sim$ common prior μ

N voters (odd)

- voter i chooses alternative $a_i \in \{1,0\}$
- simple majority rule

$$\textbf{vote share } \bar{a}_N = \frac{1}{N}\sum_{i=1}^{N} a_i \begin{cases} > 1/2 & \Longleftrightarrow \quad \text{winner} = 1 \\ < 1/2 & \Longleftrightarrow \quad \text{winner} = 0 \end{cases}$$

- common interests

$$u(\bar{a}_N, \theta) = \begin{cases} 1 & \text{if winner} = \theta \\ 0 & \text{if winner} \neq \theta \end{cases}$$

Rational Inattention Sims 2003, Matějka–McKay 2015, Denti 2023

state

θ

influence

votes

$a_1, \ldots, a_N$

influence

aggregate

poll

$\bar{a}_{-i} \equiv \frac{1}{N-1} \sum_{j \neq i} a_j$

poll = probable vote share

Voter i optimizes over conditional action distributions $P_i\left(a_i \mid \bar{a}_{-i}, \theta\right)$:

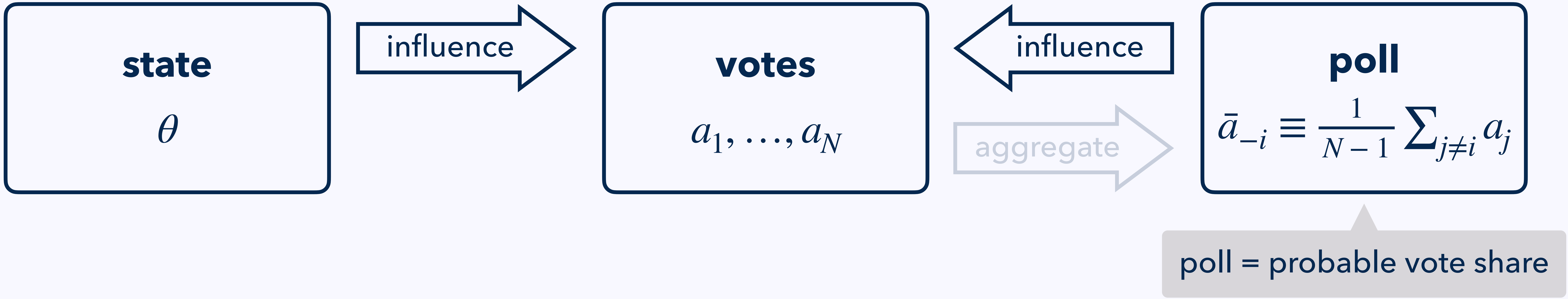
$$\max_{P_i} \; \mathbb{E}\left[u\left(\bar{a}_N, \theta\right)\right] - \text{info cost}$$

Entropy-based cost

$$\text{info cost} = \text{unit cost } \lambda \cdot \text{uncertainty reduction in terms of entropy}$$

more

Rational Inattention Sims 2003, Matějka–McKay 2015, Denti 2023



Voter i optimizes over conditional action distributions $P_i\left(a_i \mid \bar{a}_{-i}, \theta\right)$:

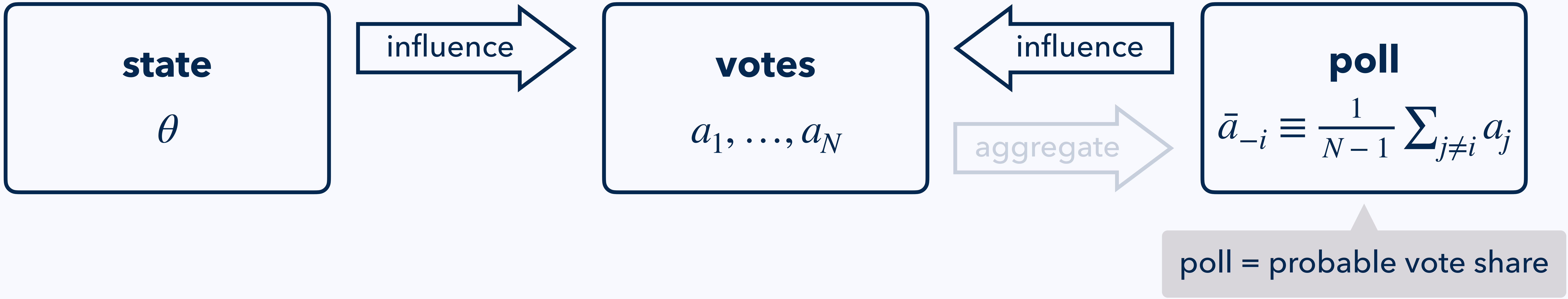
$$\max_{P_i} \; \mathbb{E}\left[u\left(\bar{a}_N, \theta\right)\right] - \text{info cost}$$

Entropy-based cost

$$\text{info cost} = \text{unit cost } \lambda \cdot \text{uncertainty reduction in terms of entropy}$$

more

Rational Inattention
Sims 2003, Matějka–McKay 2015, Denti 2023



Voter i optimizes over conditional action distributions $P_i\left(a_i \mid \bar{a}_{-i}, \theta\right)$:

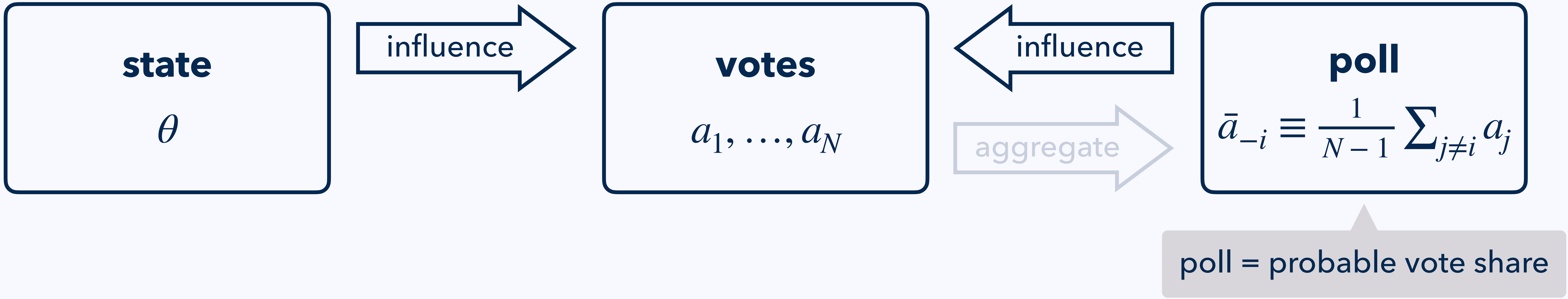
$$\max_{P_i} \ \mathbb{E}\left[u\left(\bar{a}_N, \theta\right)\right] - \text{info cost}$$

Entropy-based cost

$$\text{info cost} = \text{unit cost } \lambda \cdot \text{uncertainty reduction in terms of entropy}$$

more

Rational Inattention Sims 2003, Matějka–McKay 2015, Denti 2023



Voter i optimizes over conditional action distributions $P_i(a_i \mid \bar{a}_{-i}, \theta)$:

$$\max_{P_i} \ \mathbb{E}\left[u(\bar{a}_N, \theta)\right] - \text{info cost}$$

Entropy-based cost

$$\text{info cost} = \text{unit cost } \lambda \cdot \text{uncertainty reduction in terms of entropy}$$

$$= \text{prior entropy} - \mathbb{E}[\text{posterior entropy}]$$

more

Rational Inattention Sims 2003, Matějka–McKay 2015, Denti 2023

state θ $\xrightarrow{\text{influence}}$ **votes** $a_1, \ldots, a_N$ $\xleftarrow{\text{influence}}$ **poll** $\bar{a}_{-i} \equiv \frac{1}{N-1} \sum_{j \neq i} a_j$

votes $\xrightarrow{\text{aggregate}}$ **poll**

poll = probable vote share

Voter i optimizes over conditional action distributions $P_i(a_i \mid \bar{a}_{-i}, \theta)$:

$$\max_{P_i} \; \mathbb{E}\left[u(\bar{a}_N, \theta)\right] - \text{info cost}$$

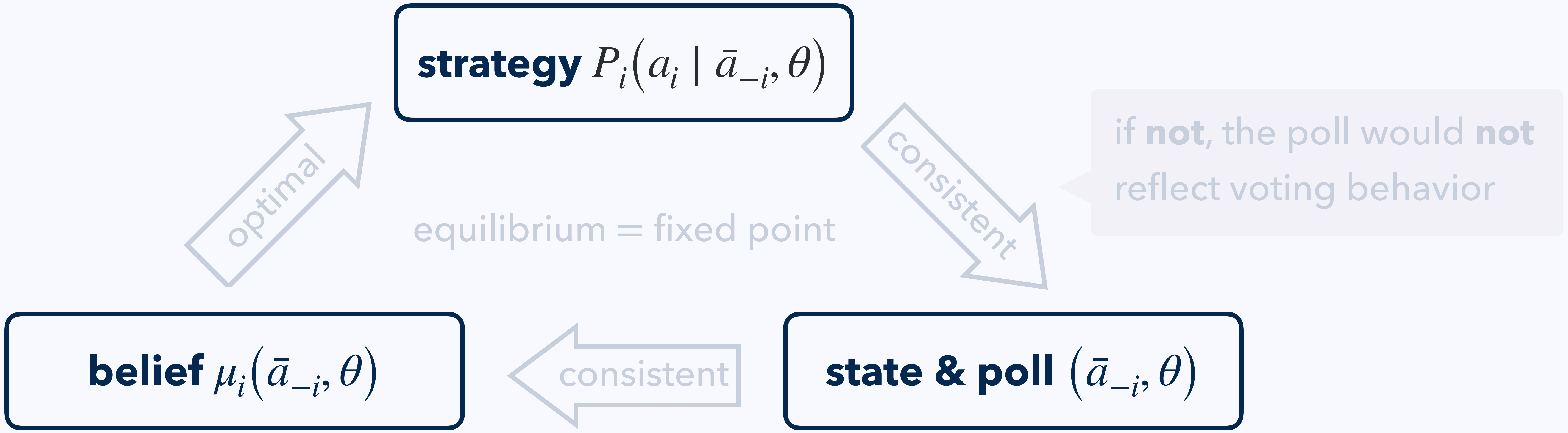
Entropy-based cost

$$\text{info cost} = \text{unit cost } \lambda \cdot \text{uncertainty reduction in terms of entropy}$$

$$= \text{prior entropy} - \mathbb{E}[\text{posterior entropy}]$$

more

Equilibrium Denti 2023, Hébert–La'O 2023

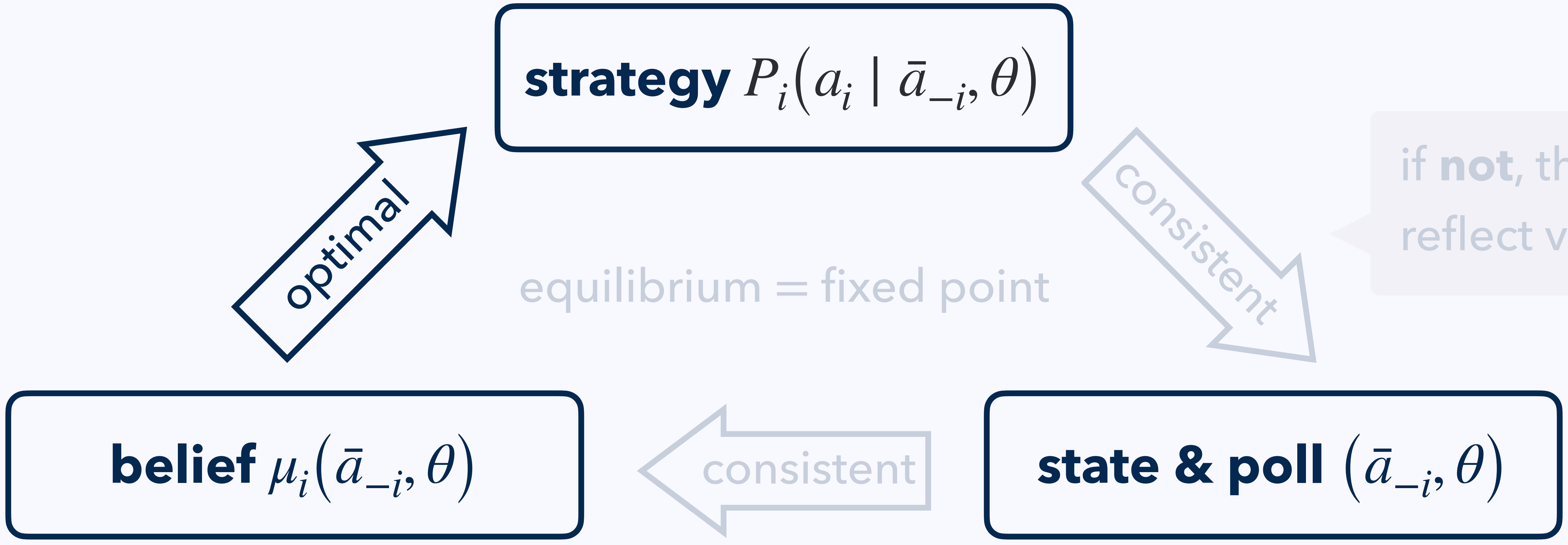


An **equilibrium** is a joint distribution $P_N^*(a_1, \ldots, a_N, \theta)$ such that:

- optimality: conditional distribution $P_N^*(a_i \mid \bar{a}_{-i}, \theta)$ = voter i's best response
- consistency: marginal distribution of state θ = prior μ

focus on symmetric equilibria

Equilibrium Denti 2023, Hébert–La'O 2023



if **not**, the poll would **not** reflect voting behavior

An **equilibrium** is a joint distribution $P_N^*(a_1, \ldots, a_N, \theta)$ such that:

- optimality: conditional distribution $P_N^*(a_i \mid \bar{a}_{-i}, \theta)$ = voter i's best response
- consistency: marginal distribution of state θ = prior μ

focus on symmetric equilibria

Equilibrium Denti 2023, Hébert–La'O 2023

strategy $P_i(a_i \mid \bar{a}_{-i}, \theta)$

optimal

consistent

if **not**, the poll would **not** reflect voting behavior

equilibrium = fixed point

belief $\mu_i(\bar{a}_{-i}, \theta)$

consistent

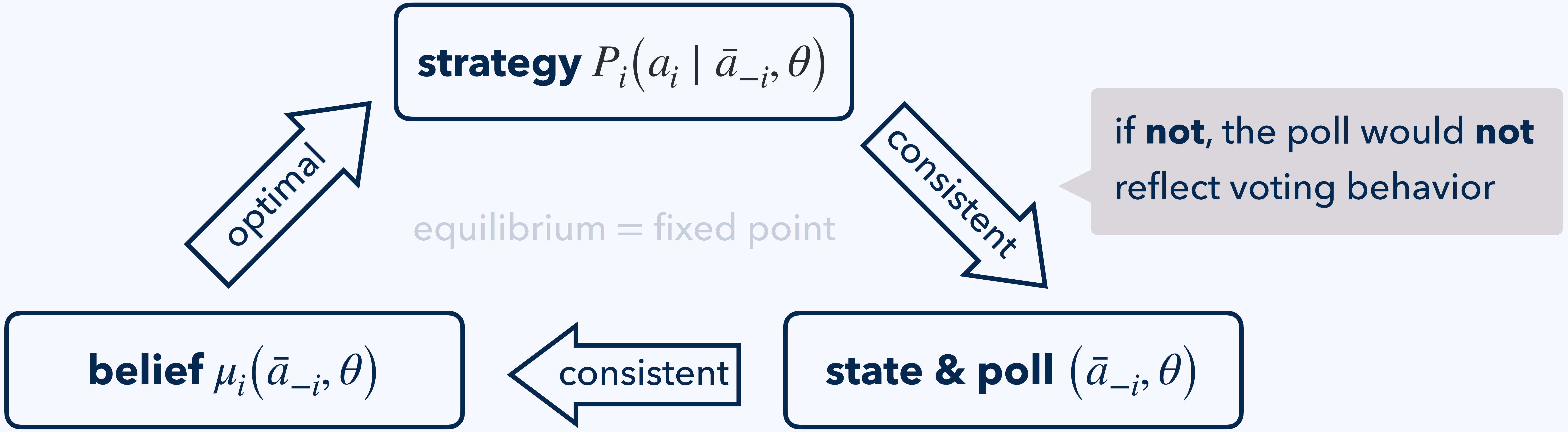
state & poll $(\bar{a}_{-i}, \theta)$

An **equilibrium** is a joint distribution $P_N^*(a_1, \ldots, a_N, \theta)$ such that:

- optimality: conditional distribution $P_N^*(a_i \mid \bar{a}_{-i}, \theta)$ = voter i's best response
- consistency: marginal distribution of state θ = prior μ

focus on symmetric equilibria

Equilibrium Denti 2023, Hébert–La'O 2023

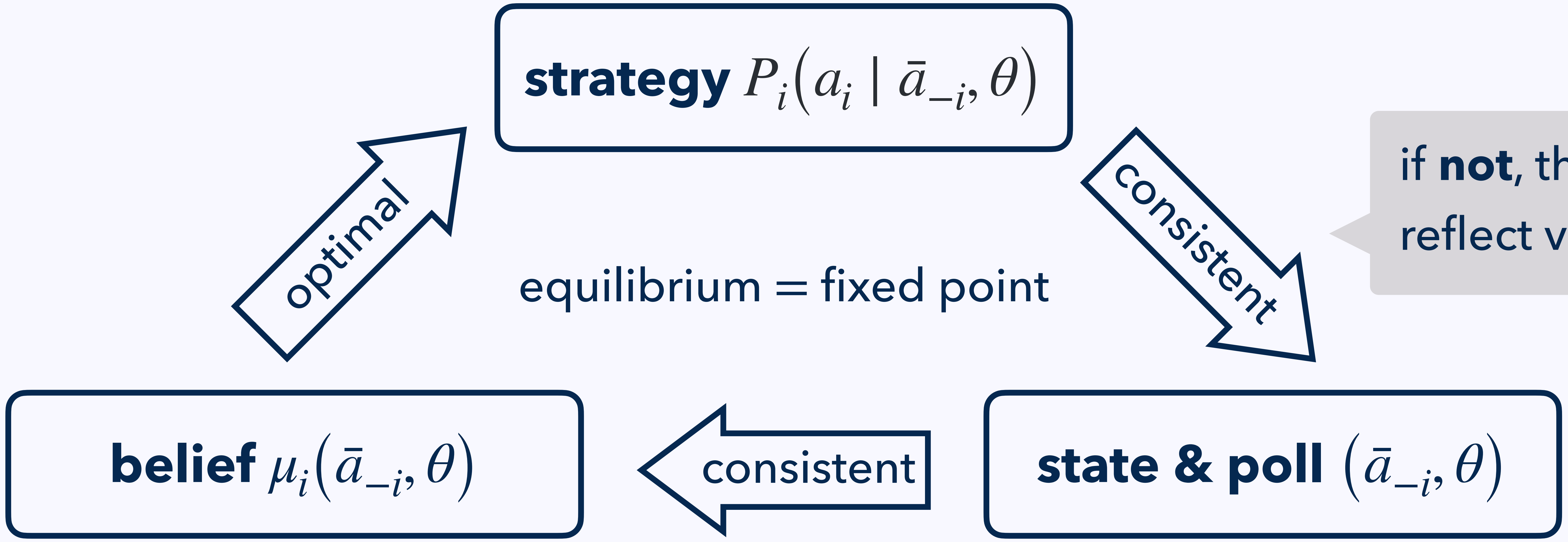


An **equilibrium** is a joint distribution $P_N^*(a_1, \dots, a_N, \theta)$ such that:

- optimality: conditional distribution $P_N^*(a_i \mid \bar{a}_{-i}, \theta)$ = voter i's best response
- consistency: marginal distribution of state θ = prior μ

focus on symmetric equilibria

Equilibrium Denti 2023, Hébert–La'O 2023



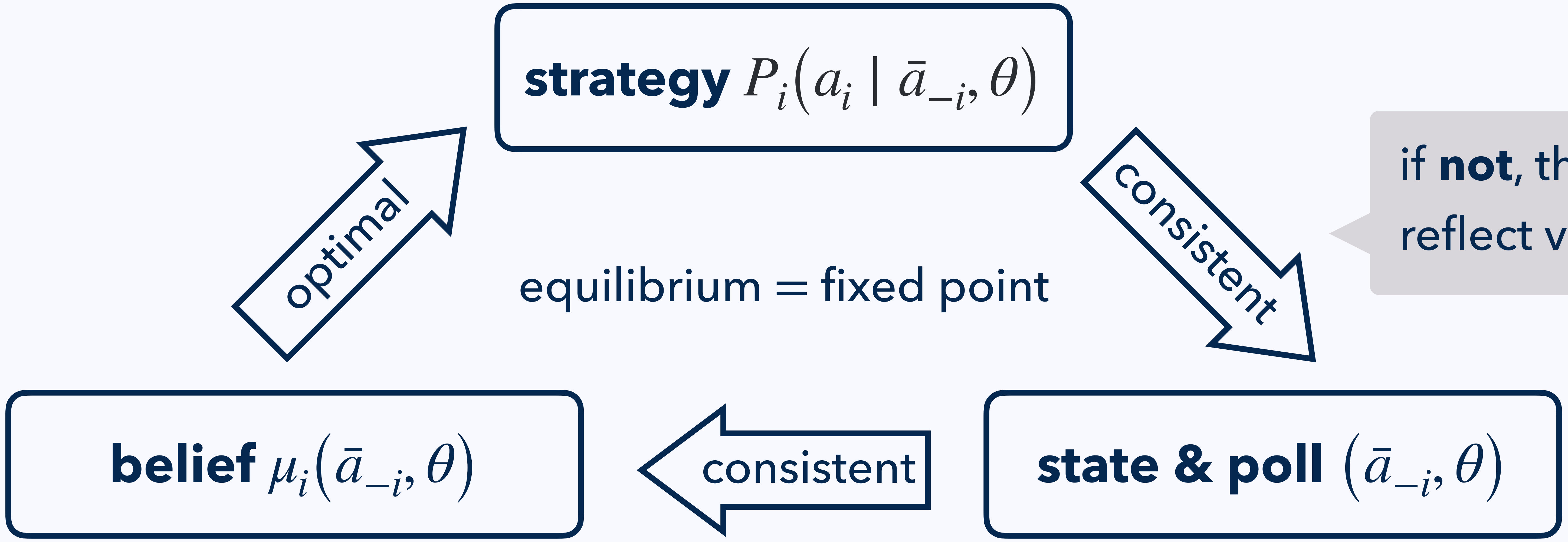
if **not**, the poll would **not** reflect voting behavior

An **equilibrium** is a joint distribution $P_N^*(a_1, \ldots, a_N, \theta)$ such that:

- optimality: conditional distribution $P_N^*(a_i \mid \bar{a}_{-i}, \theta)$ = voter i's best response
- consistency: marginal distribution of state θ = prior μ

focus on symmetric equilibria

Equilibrium Denti 2023, Hébert–La’O 2023



if **not**, the poll would **not** reflect voting behavior

An **equilibrium** is a joint distribution $P_N^*(a_1, \ldots, a_N, \theta)$ such that:

- optimality: conditional distribution $P_N^*(a_i \mid \bar{a}_{-i}, \theta)$ = voter i's best response
- consistency: marginal distribution of state $\theta = $ prior μ

focus on symmetric equilibria

Interpretations Denti 2023, Hébert–La'O 2023

A blend of two equilibrium concepts

- Bayesian Nash equilibrium
  - voters optimize under uncertainty
- rational-expectations equilibrium
  - voters condition their behavior upon endogenous aggregate votes (= poll)

Grossman–Stiglitz's rational expectations equilibrium

- agents condition their behavior upon endogenous aggregate behavior (e.g., prices, polls)

Dynamic electoral processes:

- start with any profile $a^{(0)} = \left(a_1^{(0)}, \dots, a_N^{(0)}\right)$
- randomly chosen voter i learns about $\left(\theta, \bar{a}_N^{(t-1)}\right)$ and may revise action $a_i^{(t)}$ myopically
  - any revision is reflected in the new poll $\bar{a}_N^{(t)}$
- stationary distribution = equilibrium P_N^*

Interpretations Denti 2023, Hébert–La'O 2023

A blend of two equilibrium concepts

- Bayesian Nash equilibrium
  - voters optimize under uncertainty
- rational-expectations equilibrium
  - voters condition their behavior upon endogenous aggregate votes (= poll)

Grossman–Stiglitz's rational expectations equilibrium

- agents condition their behavior upon endogenous aggregate behavior (e.g., prices, polls)

Dynamic electoral processes:

- start with any profile $a^{(0)} = \left(a_1^{(0)}, \ldots, a_N^{(0)}\right)$
- randomly chosen voter i learns about $\left(\theta, \bar{a}_N^{(t-1)}\right)$ and may revise action $a_i^{(t)}$ myopically
  - any revision is reflected in the new poll $\bar{a}_N^{(t)}$
- stationary distribution = equilibrium P_N^*

Interpretations Denti 2023, Hébert–La'O 2023

A blend of two equilibrium concepts

- Bayesian Nash equilibrium
  - voters optimize under uncertainty
- rational-expectations equilibrium
  - voters condition their behavior upon endogenous aggregate votes (= poll)

Grossman–Stiglitz's rational expectations equilibrium

- agents condition their behavior upon endogenous aggregate behavior (e.g., prices, polls)

Dynamic electoral processes:

- start with any profile $a^{(0)} = \left(a_1^{(0)}, \ldots, a_N^{(0)}\right)$
- randomly chosen voter i learns about $\left(\theta, \bar{a}_N^{(t-1)}\right)$ and may revise action $a_i^{(t)}$ myopically
  - any revision is reflected in the new poll $\bar{a}_N^{(t)}$
- stationary distribution = equilibrium P_N^*

Interpretations Denti 2023, Hébert–La'O 2023

A blend of two equilibrium concepts

- Bayesian Nash equilibrium
  - voters optimize under uncertainty
- rational-expectations equilibrium
  - voters condition their behavior upon endogenous aggregate votes (= poll)

Grossman–Stiglitz's rational expectations equilibrium

- agents condition their behavior upon endogenous aggregate behavior (e.g., prices, polls)

Dynamic electoral processes:

- start with any profile $a^{(0)} = \left(a_1^{(0)}, \ldots, a_N^{(0)}\right)$
- randomly chosen voter i learns about $\left(\theta, \bar{a}_N^{(t-1)}\right)$ and may revise action $a_i^{(t)}$ myopically
  - any revision is reflected in the new poll $\bar{a}_N^{(t)}$
- stationary distribution = equilibrium P_N^*

Interpretations Denti 2023, Hébert–La'O 2023

A blend of two equilibrium concepts

- Bayesian Nash equilibrium
  - voters optimize under uncertainty
- rational-expectations equilibrium
  - voters condition their behavior upon endogenous aggregate votes (= poll)

Grossman–Stiglitz's rational expectations equilibrium
- agents condition their behavior upon endogenous aggregate behavior (e.g., prices, polls)

Dynamic electoral processes:

- start with any profile $a^{(0)} = \left(a_1^{(0)}, \ldots, a_N^{(0)}\right)$
- randomly chosen voter i learns about $\left(\theta, \bar{a}_N^{(t-1)}\right)$ and may revise action $a_i^{(t)}$ myopically
  - any revision is reflected in the new poll $\bar{a}_N^{(t)}$
- stationary distribution = equilibrium P_N^*

Interpretations Denti 2023, Hébert–La'O 2023

A blend of two equilibrium concepts

- Bayesian Nash equilibrium
  - voters optimize under uncertainty
- rational-expectations equilibrium
  - voters condition their behavior upon endogenous aggregate votes (= poll)

Grossman–Stiglitz's rational expectations equilibrium
- agents condition their behavior upon endogenous aggregate behavior (e.g., prices, polls)

Dynamic electoral processes:

- start with any profile $a^{(0)} = \left(a_1^{(0)}, \ldots, a_N^{(0)}\right)$
- randomly chosen voter i learns about $\left(\theta, \bar{a}_N^{(t-1)}\right)$ and may revise action $a_i^{(t)}$ myopically
  - any revision is reflected in the new poll $\bar{a}_N^{(t)}$
- stationary distribution = equilibrium P_N^*

Interpretations Denti 2023, Hébert–La'O 2023

A blend of two equilibrium concepts

- Bayesian Nash equilibrium
  - voters optimize under uncertainty
- rational-expectations equilibrium
  - voters condition their behavior upon endogenous aggregate votes (= poll)

Grossman–Stiglitz's rational expectations equilibrium

- agents condition their behavior upon endogenous aggregate behavior (e.g., prices, polls)

Dynamic electoral processes:

- start with any profile $a^{(0)} = \left(a_1^{(0)}, \ldots, a_N^{(0)}\right)$
- randomly chosen voter i learns about $\left(\theta, \bar{a}_N^{(t-1)}\right)$ and may revise action $a_i^{(t)}$ myopically
  - any revision is reflected in the new poll $\bar{a}_N^{(t)}$
- stationary distribution = equilibrium P_N^*

Every Equilibrium Is Biased-Logit Matějka–McKay 2015, Denti 2023

p_N^* = marginal probability $(a_i = 1)$

$$\Pr\left(\bar{a}_N = \frac{k}{N} \mid \theta\right) = \frac{1}{Z_N(p_N^*, \theta)} \binom{N}{k} \exp\left[\frac{u\left(\frac{k}{N}, \theta\right)}{\lambda}\right] (p_N^*)^k (1 - p_N^*)^{N-k}$$

$Z_N(p_N^*, \theta)$ = normalizing constant

- **informative equilibrium** in which voters acquire information
  - it **exists** if and only if $e^{-1/\lambda} < \frac{\mu(1)}{\mu(0)} < e^{1/\lambda}$, and it is **unique** whenever it exists
- uninformative equilibrium in which voters acquire no information
  - all choose alternative 1
  - all choose alternative 0

more

Every Equilibrium Is Biased-Logit

Matějka–McKay 2015, Denti 2023

p_N^* = marginal probability $(a_i = 1)$

$$\Pr\left(\bar{a}_N = \frac{k}{N} \mid \theta\right) = \frac{1}{Z_N(p_N^*, \theta)} \binom{N}{k} \exp\left[\frac{u\left(\frac{k}{N}, \theta\right)}{\lambda}\right] (p_N^*)^k (1 - p_N^*)^{N-k}$$

$Z_N(p_N^*, \theta)$ = normalizing constant

- **informative equilibrium** in which voters acquire information
  - it **exists** if and only if $e^{-1/\lambda} < \frac{\mu(1)}{\mu(0)} < e^{1/\lambda}$, and it is **unique** whenever it exists
- uninformative equilibrium in which voters acquire no information
  - all choose alternative 1
  - all choose alternative 0

more

Every Equilibrium Is Biased-Logit Matějka–McKay 2015, Denti 2023

p_N^* = marginal probability $(a_i = 1)$

$$\Pr\left(\bar{a}_N = \frac{k}{N} \mid \theta\right) = \frac{1}{Z_N(p_N^*, \theta)} \binom{N}{k} \exp\left[\frac{u\left(\frac{k}{N}, \theta\right)}{\lambda}\right] \left(p_N^*\right)^k \left(1 - p_N^*\right)^{N-k}$$

$Z_N(p_N^*, \theta)$ = normalizing constant

- **informative equilibrium** in which voters acquire information
  - it **exists** if and only if $e^{-1/\lambda} < \frac{\mu(1)}{\mu(0)} < e^{1/\lambda}$, and it is **unique** whenever it exists
- uninformative equilibrium in which voters acquire no information
  - all choose alternative 1
  - all choose alternative 0

more

Every Equilibrium Is Biased-Logit Matějka–McKay 2015, Denti 2023

p_N^* = marginal probability $(a_i = 1)$

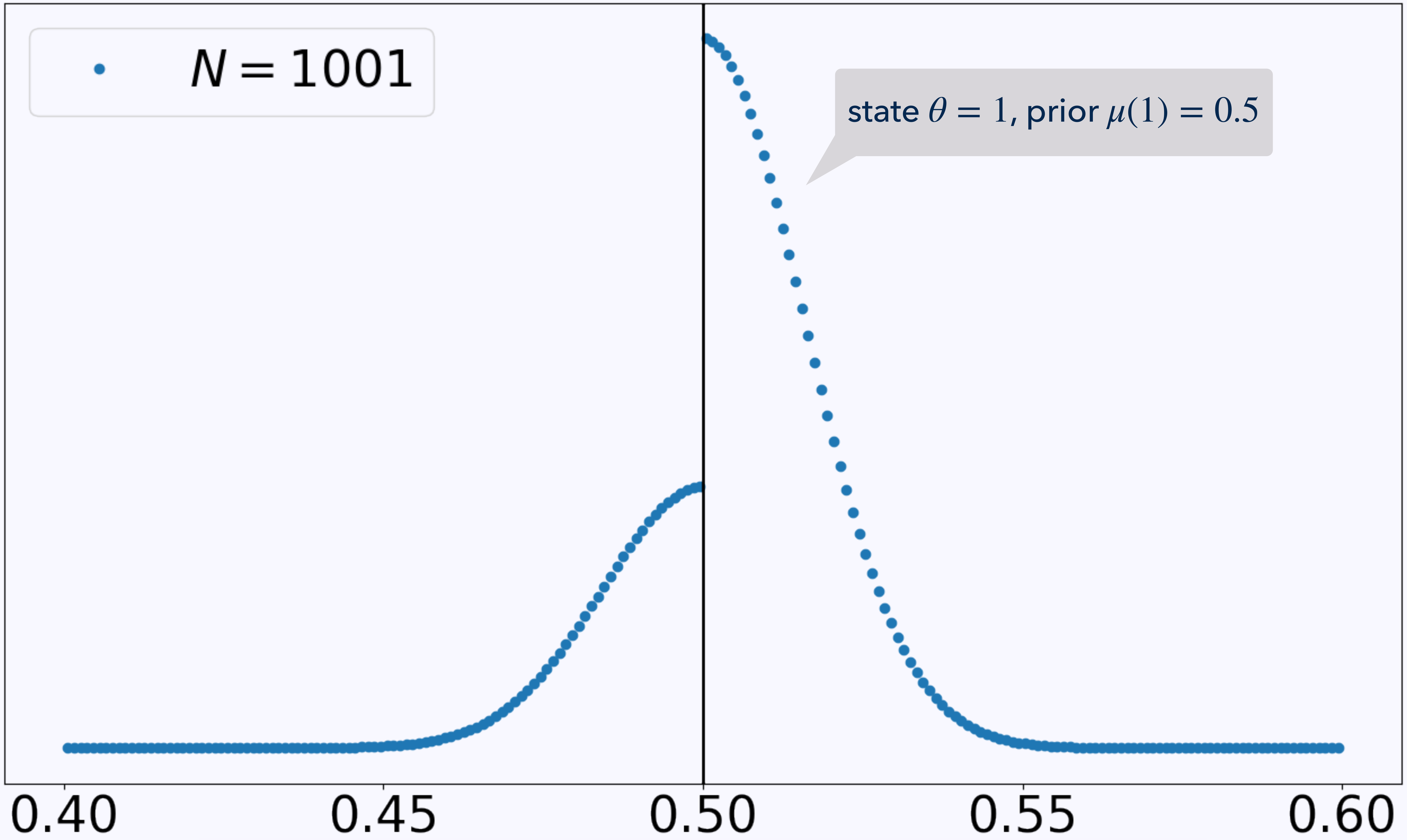
$$\Pr\left(\bar{a}_N = \frac{k}{N} \mid \theta\right) = \frac{1}{Z_N\left(p_N^*, \theta\right)} \binom{N}{k} \exp\left[\frac{u\left(\frac{k}{N}, \theta\right)}{\lambda}\right] \left(p_N^*\right)^k \left(1 - p_N^*\right)^{N-k}$$

$Z_N\left(p_N^*, \theta\right)$ = normalizing constant

- **informative equilibrium** in which voters acquire information
  - it **exists** if and only if $e^{-1/\lambda} < \dfrac{\mu(1)}{\mu(0)} < e^{1/\lambda}$, and it is **unique** whenever it exists
- uninformative equilibrium in which voters acquire no information
  - all choose alternative 1
  - all choose alternative 0

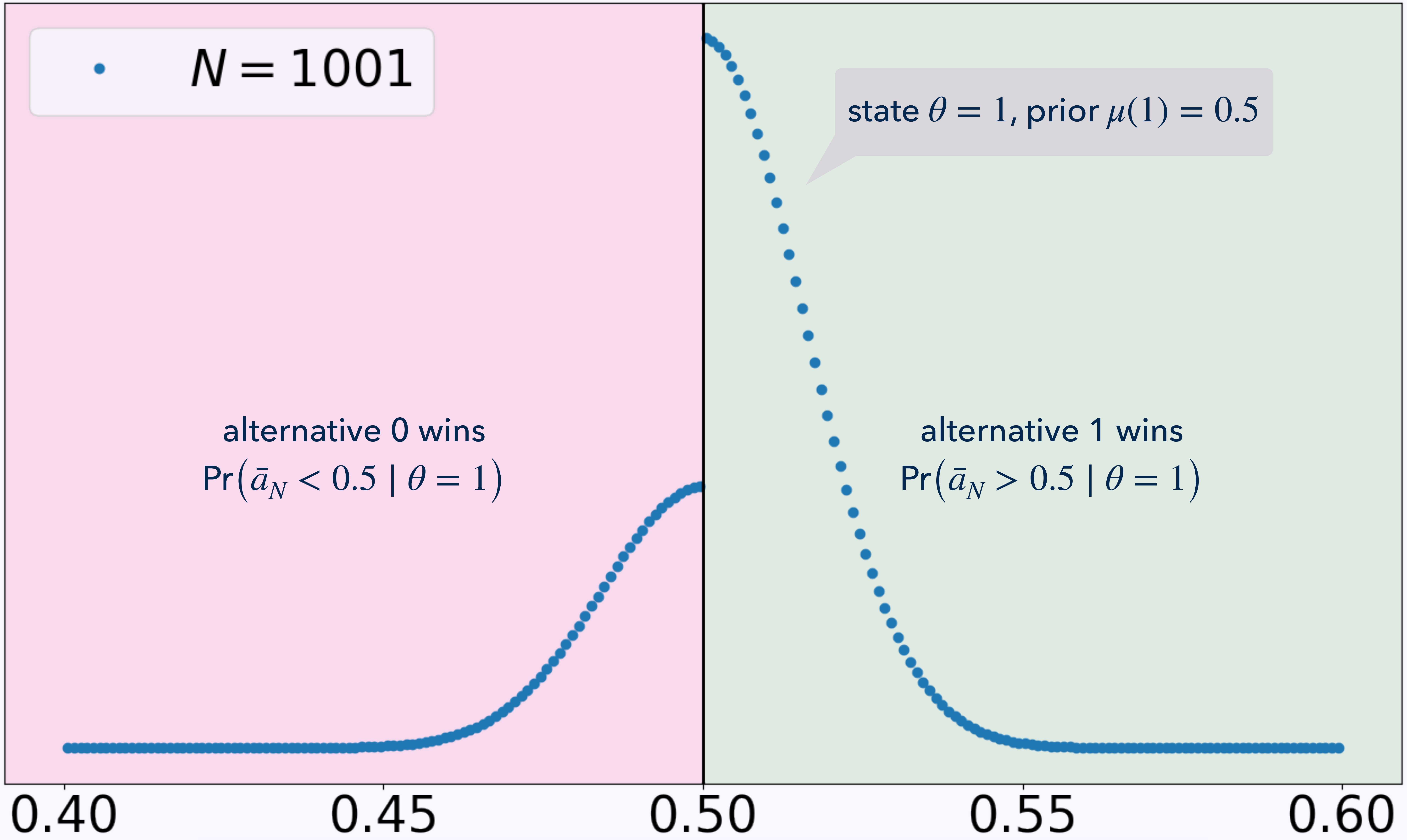
more

Vote-Share Distribution



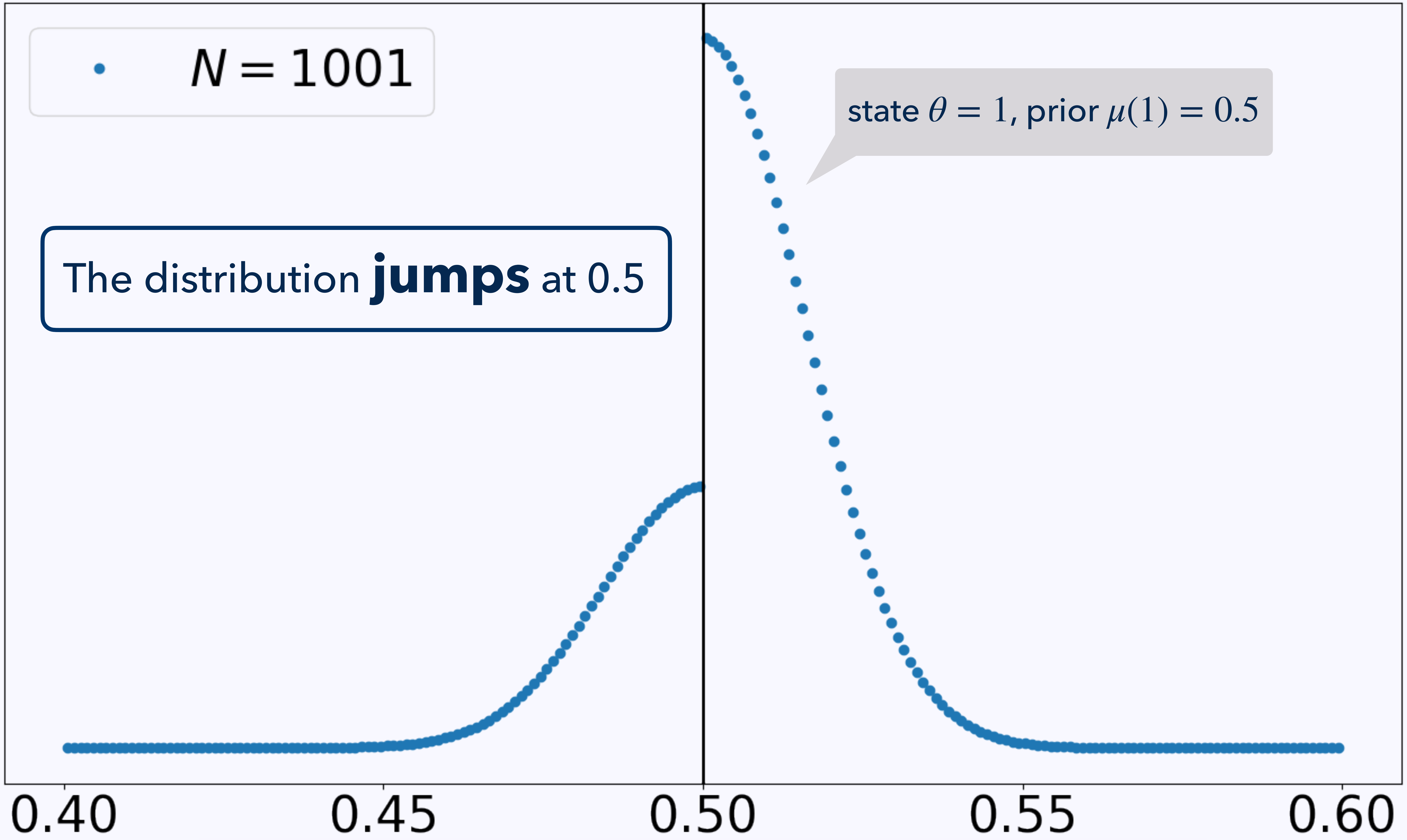
x-axis: vote share $\bar{a}_N = x$ y-axis: $\Pr(\bar{a}_N = x \mid \theta = 1)$

Vote-Share Distribution



x-axis: vote share $\bar{a}_N = x$ y-axis: $\Pr(\bar{a}_N = x \mid \theta = 1)$

Vote-Share Distribution



x-axis: vote share $\bar{a}_N = x$ y-axis: $\Pr(\bar{a}_N = x \mid \theta = 1)$

Probability of Correct Choice

How does the **probability of correct choice** change as #voters N increases?

$$\Pr\big(\text{correct}\big) = \mu(0)\,\underbrace{\Pr\big(\bar{a}_N < 0.5 \mid \theta = 0\big)}_{\text{choosing 0 at state 0}} + \mu(1)\,\underbrace{\Pr\big(\bar{a}_N > 0.5 \mid \theta = 1\big)}_{\text{choosing 1 at state 1}}$$

Probability of Correct Choice

How does the **probability of correct choice** change as #voters N increases?

$$\Pr\big(\text{correct}\big) = \mu(0)\,\boxed{\Pr\big(\bar{a}_N < 0.5 \mid \theta = 0\big)} + \mu(1)\,\boxed{\Pr\big(\bar{a}_N > 0.5 \mid \theta = 1\big)}$$

choosing 0 at state 0

choosing 1 at state 1

Theorem 1

The probability of correct choice is **independent of #voters N and prior μ**

$$\Pr(\text{correct}) = \frac{e^{1/\lambda}}{1 + e^{1/\lambda}}$$

proof video

Analogy to public-good provision

- the total amount of a public good is independent of #agents
- information is a public good in elections

Theorem 1

The probability of correct choice is **independent of #voters N and prior μ**

$$\mathrm{Pr}\left(\mathrm{correct}\right) = \frac{\mathrm{e}^{1/\lambda}}{1 + \mathrm{e}^{1/\lambda}}$$

proof **video**

Analogy to public-good provision

- the total amount of a public good is independent of #agents
- information is a public good in elections

Extensions

Elections with **supermajority rule**

unanimity rule if $\alpha = 1$

- alternative 1 wins $\iff$ vote share $\bar{a}_N \geq$ threshold $\alpha \in (0.5, 1]$
- Pr(correct) is **independent of the threshold** α

$$\Pr\left(\text{correct}\right) = \frac{\mathrm{e}^{1/\lambda}}{1 + \mathrm{e}^{1/\lambda}}$$

more

Elections with $N_1, N_0 < N/2$ **partisans**, who always vote for their favorite

- the others are non-partisans, who want to choose the correct alternative θ
- Pr(correct) is **independent of #partisans** N_1, N_0

$$\Pr\left(\text{correct}\right) = \frac{\mathrm{e}^{1/\lambda}}{1 + \mathrm{e}^{1/\lambda}}$$

more

Extensions

Elections with **supermajority rule**

unanimity rule if $\alpha = 1$

- alternative 1 wins $\iff$ vote share $\bar{a}_N \geq$ threshold $\alpha \in (0.5, 1]$
- $\Pr(\text{correct})$ is **independent of the threshold** α

$$\Pr(\text{correct}) = \frac{e^{1/\lambda}}{1 + e^{1/\lambda}}$$

more

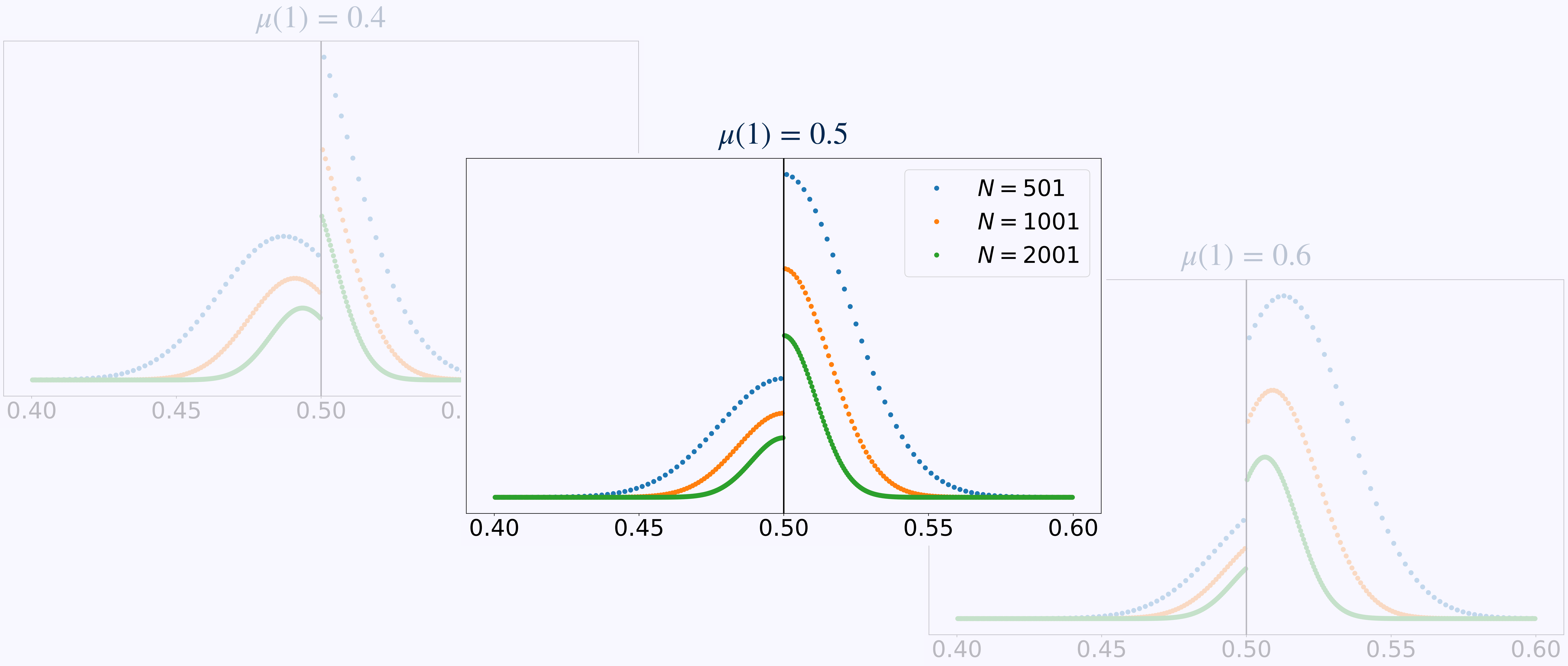
Elections with $N_1, N_0 < N/2$ **partisans**, who always vote for their favorite

- the others are non-partisans, who want to choose the correct alternative θ
- $\Pr(\text{correct})$ is **independent of #partisans** N_1, N_0

$$\Pr(\text{correct}) = \frac{e^{1/\lambda}}{1 + e^{1/\lambda}}$$

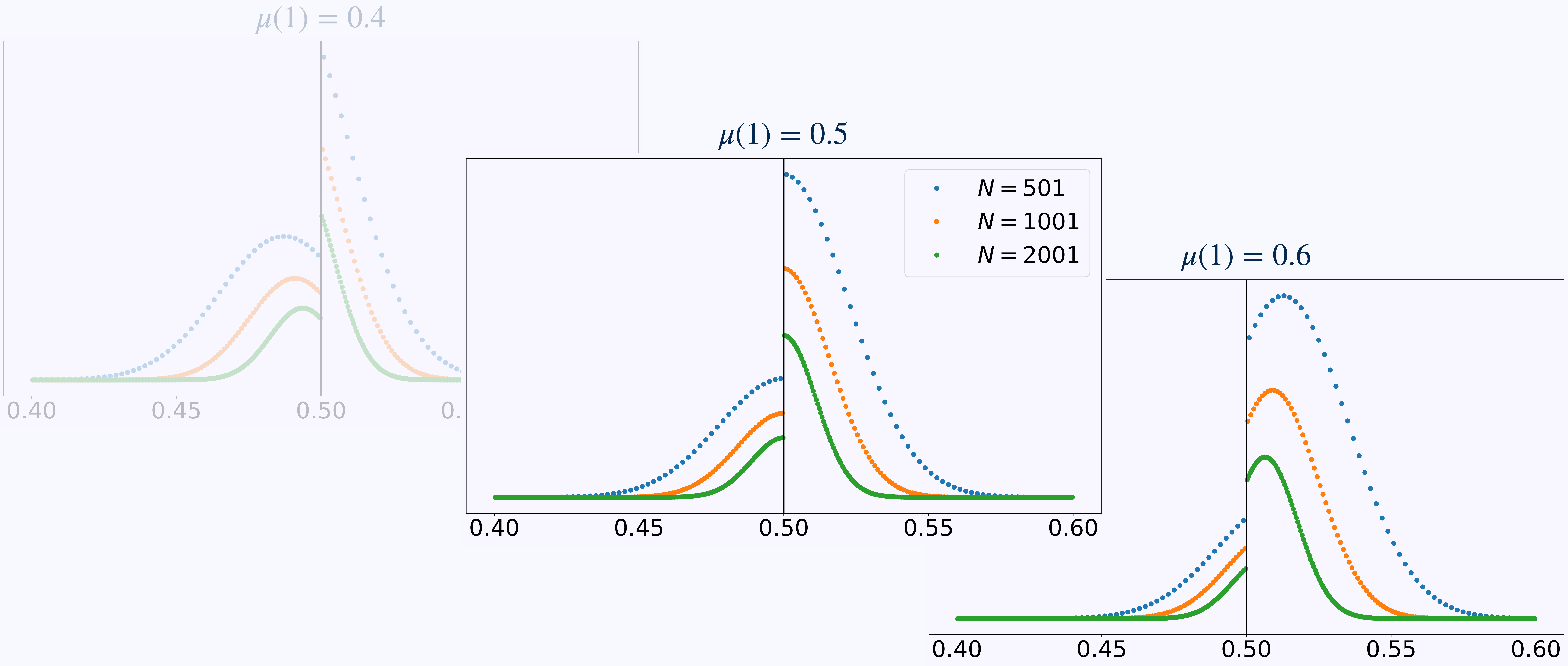
more

Vote-Share Concentration



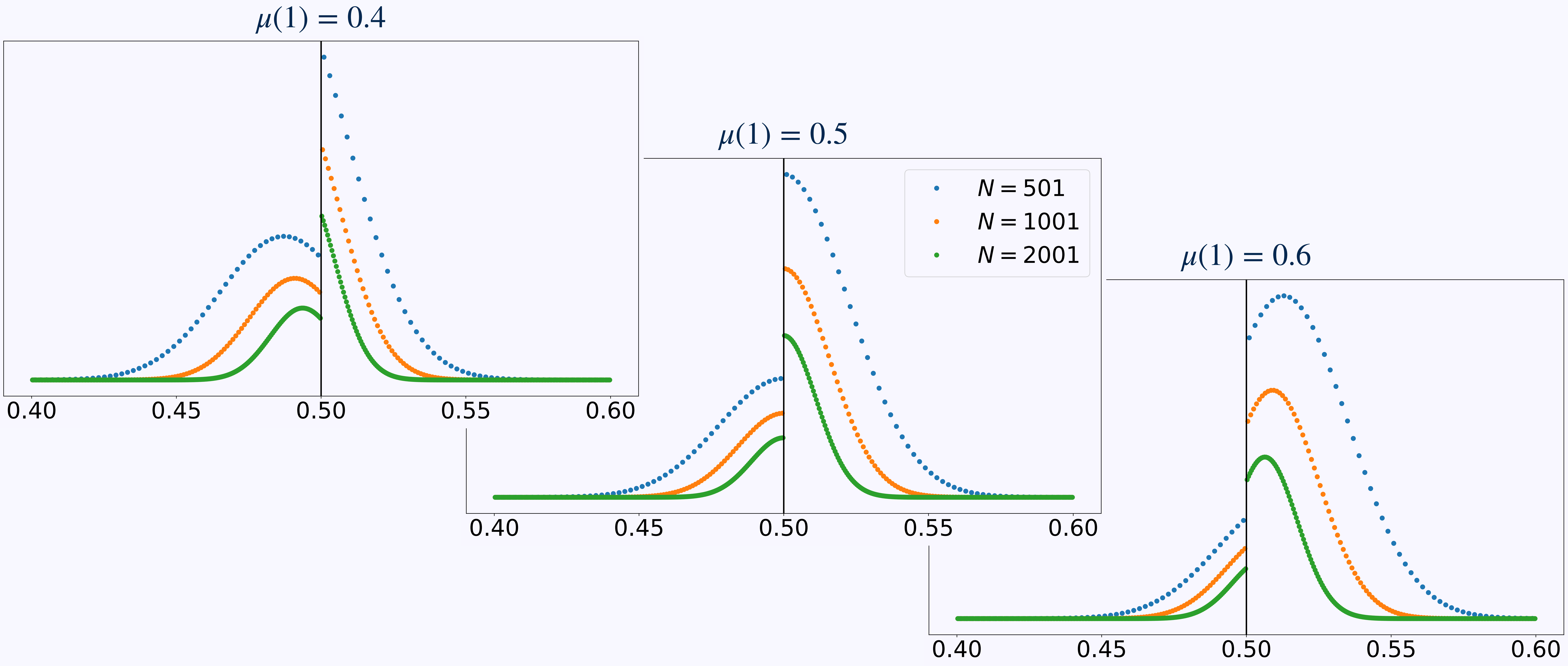
The vote shares are **concentrated on the threshold 0.5** even if prior $\mu(1) \neq 0.5$

Vote-Share Concentration



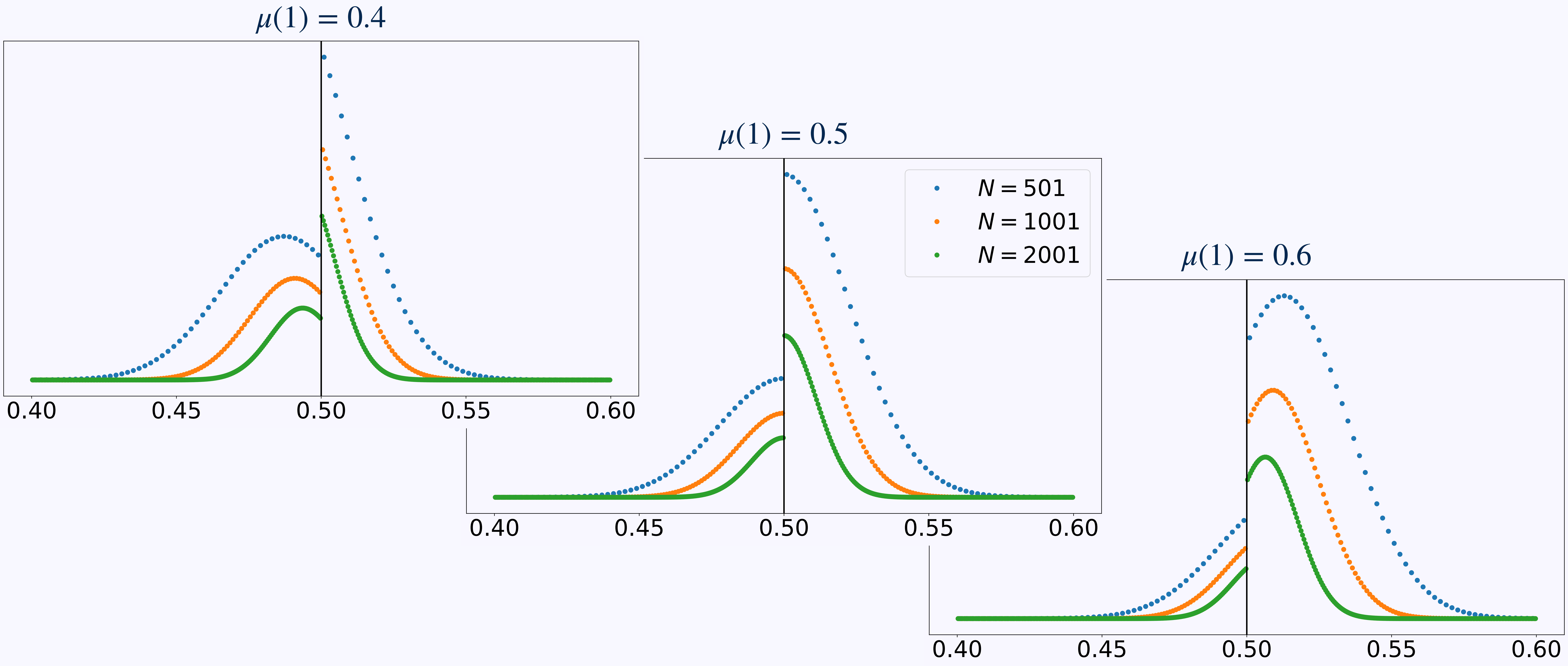
The vote shares are **concentrated on the threshold 0.5** even if prior $\mu(1) \neq 0.5$

Vote-Share Concentration



The vote shares are **concentrated on the threshold 0.5** even if prior $\mu(1) \neq 0.5$

Vote-Share Concentration



The vote shares are **concentrated on the threshold 0.5** even if prior $\mu(1) \neq 0.5$

Theorem 2

Large elections will be **close regardless of prior μ and unit cost λ**

$$\Pr\left(\left|\bar{a}_N - 0.5\right| < \varepsilon\right) \xrightarrow{N\to\infty} 1 \quad \forall\varepsilon > 0$$

proof video

Proof sketch

- marginal probability $p_N^* \xrightarrow{N\to\infty} 0.5$
  - each voter is less likely to be pivotal and wants to free-ride on others who may have useful information
  - the best way to free-ride is to cast a neutral vote (not to dilute information)
- bound binomial tail for nearly 50-50 independent votes

more

Extensions

unanimity rule if $\alpha = 1$

Elections with **supermajority rule**

- alternative 1 wins $\iff$ vote share $\bar{a}_N \geq$ threshold $\alpha \in (0.5, 1]$
- large elections are **close regardless of threshold** α:

$$\Pr\left(\left| \bar{a}_N - \alpha \right| < \varepsilon \right) \xrightarrow{N \to \infty} 1$$

Elections with $N_1, N_0 < N/2$ **partisans**, who always vote for their favorite

- the others are non-partisans, who want to choose the correct alternative θ
- large elections are **close regardless of #partisans** N_1, N_0:

$$\Pr\left(\left| \bar{a}_N - 0.5 \right| < \varepsilon \right) \xrightarrow{N \to \infty} 1$$

more

Extensions

Elections with **supermajority rule**

unanimity rule if $\alpha = 1$

- alternative 1 wins $\iff$ vote share $\bar{a}_N \geq$ threshold $\alpha \in (0.5, 1]$
- large elections are **close regardless of threshold** α:

$$\Pr\left(\left| \bar{a}_N - \alpha \right| < \varepsilon \right) \xrightarrow{N\to\infty} 1$$

Elections with $N_1, N_0 < N/2$ **partisans**, who always vote for their favorite

- the others are non-partisans, who want to choose the correct alternative θ
- large elections are **close regardless of #partisans** N_1, N_0:

$$\Pr\left(\left| \bar{a}_N - 0.5 \right| < \varepsilon \right) \xrightarrow{N\to\infty} 1$$

more

Elections without Opinion Polls

with polls, actions a_i and strategies P_i
without polls, actions b_i and strategies Q_i

Voters acquire information only about state θ

- voter i optimizes over conditional action distributions $Q_i(b_i \mid \theta)$

$$\max_{Q_i} \quad \mathbb{E}\left[u\left(\bar{b}_N, \theta\right)\right] - \text{info cost}$$

An equilibrium is a joint distribution $Q_N^*\left(b_1, \ldots, b_N, \theta\right) = \mu(\theta) \prod_i Q_N^*\left(b_i \mid \theta\right)$ such that:

- conditional distribution $Q_N^*\left(b_i \mid \theta\right) =$ voter i's best response

focus on symmetric equilibria

Elections without Opinion Polls

with polls, actions a_i and strategies P_i
without polls, actions b_i and strategies Q_i

Voters acquire information only about state θ

- voter i optimizes over conditional action distributions $Q_i(b_i \mid \theta)$

$$\max_{Q_i} \quad \mathbb{E}\left[u(\bar{b}_N, \theta)\right] - \text{info cost}$$

An equilibrium is a joint distribution $Q_N^*(b_1, \ldots, b_N, \theta) = \mu(\theta)\prod_i Q_N^*(b_i \mid \theta)$ such that:

- conditional distribution $Q_N^*(b_i \mid \theta) =$ voter i's best response

focus on symmetric equilibria

with polls, actions a_i and strategies P_i
without polls, actions b_i and strategies Q_i

Voters acquire information only about state θ

- voter i optimizes over conditional action distributions $Q_i(b_i \mid \theta)$

$$\max_{Q_i} \quad \mathbb{E}\left[u\left(\bar{b}_N, \theta\right)\right] - \text{info cost}$$

An equilibrium is a joint distribution $Q_N^*\left(b_1, \ldots, b_N, \theta\right) = \mu(\theta) \prod_i Q_N^*\left(b_i \mid \theta\right)$ such that:

- conditional distribution $Q_N^*\left(b_i \mid \theta\right) =$ voter i's best response

focus on symmetric equilibria

Elections without Opinion Polls

with polls, actions a_i and strategies P_i
without polls, actions b_i and strategies Q_i

Voters acquire information only about state θ

- voter i optimizes over conditional action distributions $Q_i(b_i \mid \theta)$

$$\max_{Q_i} \quad \mathbb{E}\left[u\left(\bar{b}_N, \theta\right)\right] - \text{info cost}$$

An equilibrium is a joint distribution $Q_N^*(b_1, \ldots, b_N, \theta) = \mu(\theta) \prod_i Q_N^*(b_i \mid \theta)$ such that:

- conditional distribution $Q_N^*(b_i \mid \theta) =$ voter i's best response

focus on symmetric equilibria

Elections without Opinion Polls

with polls, actions a_i and strategies P_i
without polls, actions b_i and strategies Q_i

Voters acquire information only about state θ

- voter i optimizes over conditional action distributions $Q_i(b_i \mid \theta)$

$$\max_{Q_i} \quad \mathbb{E}\left[u\left(\bar{b}_N, \theta\right)\right] - \text{info cost}$$

An equilibrium is a joint distribution $Q_N^*(b_1, \ldots, b_N, \theta) = \mu(\theta) \prod_i Q_N^*(b_i \mid \theta)$ such that:

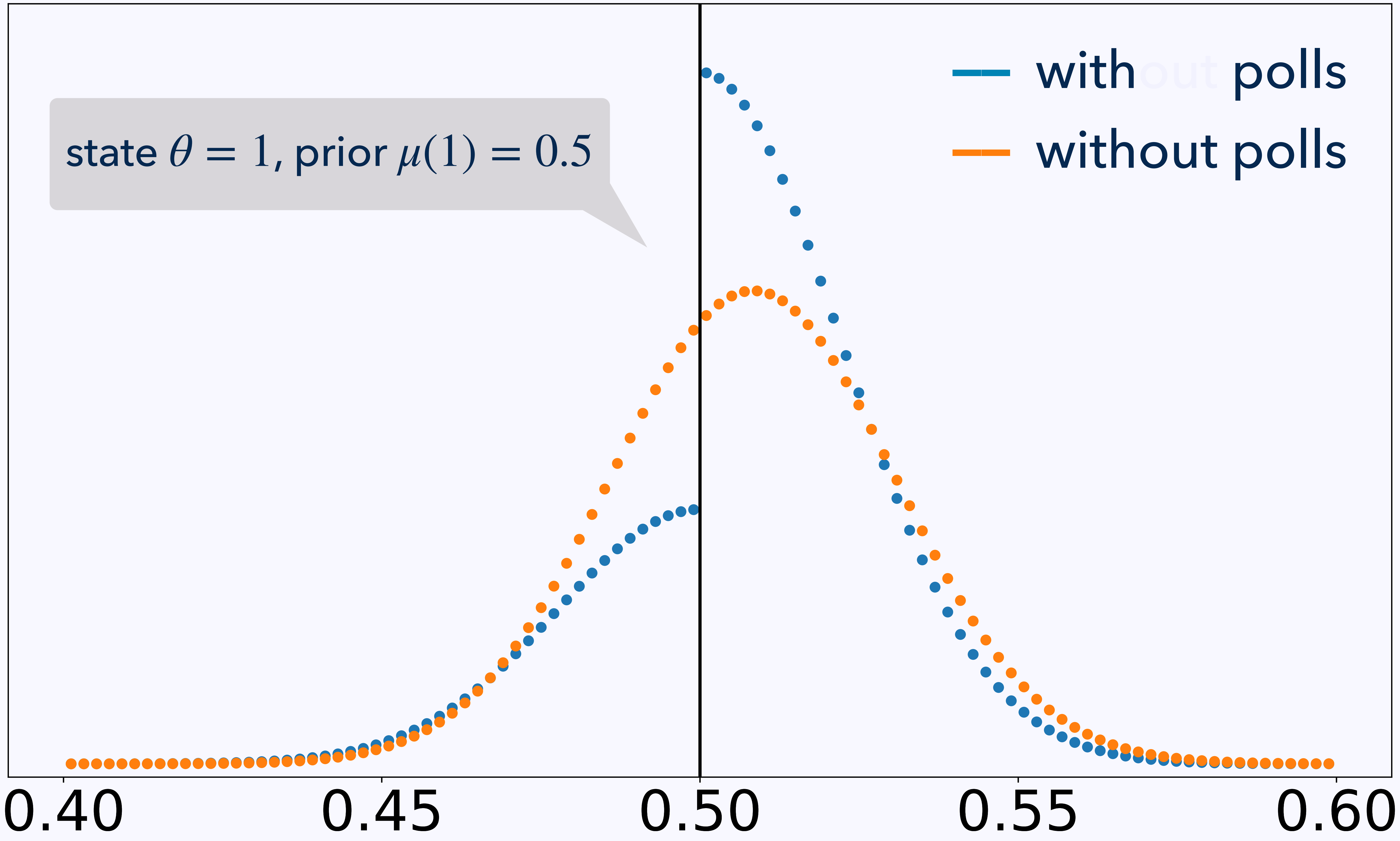
- conditional distribution $Q_N^*(b_i \mid \theta) =$ voter i's best response

focus on symmetric equilibria

Vote-Share Distributions

Without polls, the vote-share distribution **jumps** at the threshold of 0.5

Without polls, the vote-share distribution **shifts** toward the correct alternative



x-axis: vote share x y-axis: $\Pr(x \mid \theta = 1)$

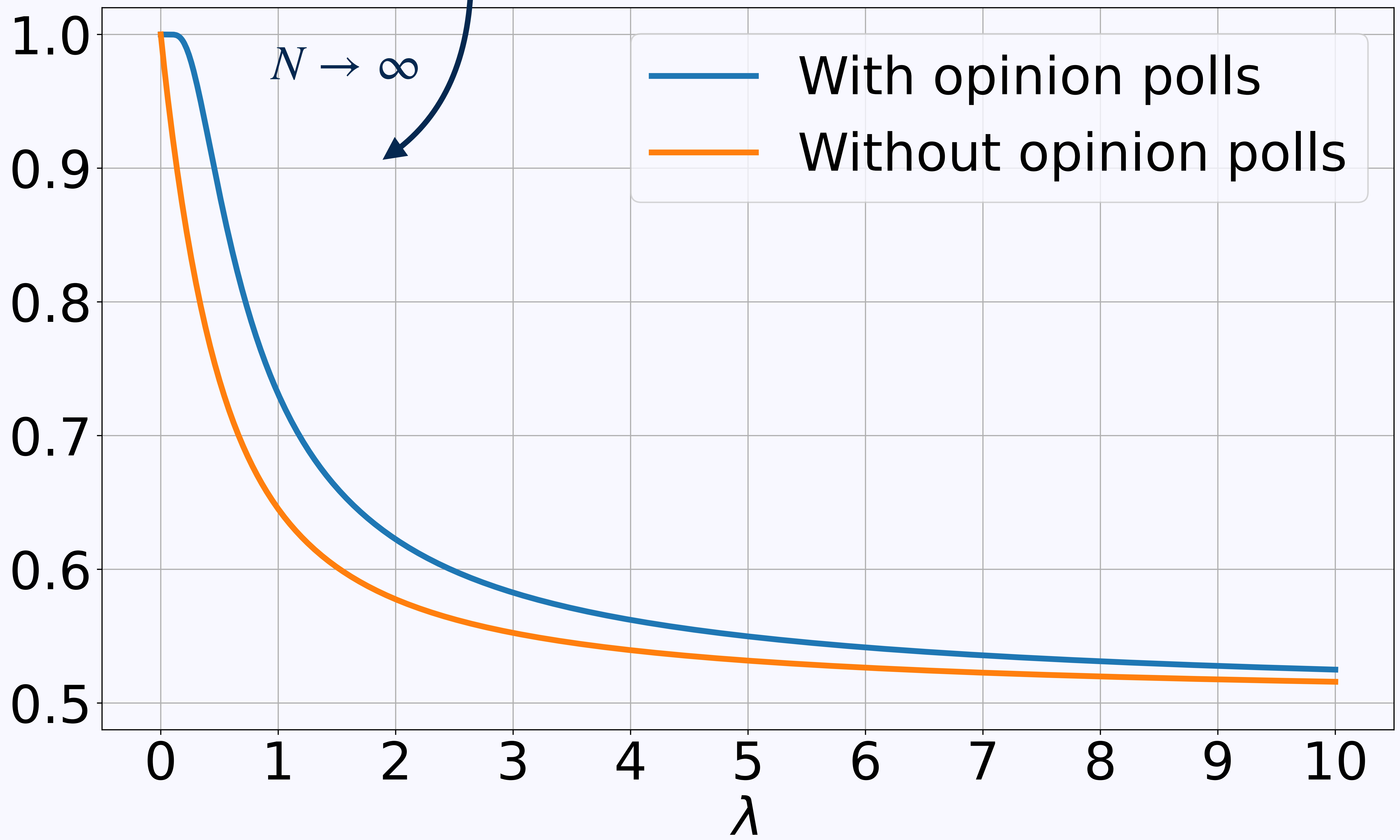
Elections with and without Opinion Polls

For the prior $\mu(1) \approx 0.5$ and any large #voters N, the probability of correct choice in the informative equilibrium with a poll is strictly higher than any equilibrium without a poll:

$$\frac{e^{1/\lambda}}{1+e^{1/\lambda}} = \Pr\big(\text{correct} \mid \text{with polls}\big) > \Pr\big(\text{correct} \mid \text{without polls}\big)$$

proof

$N \to \infty$

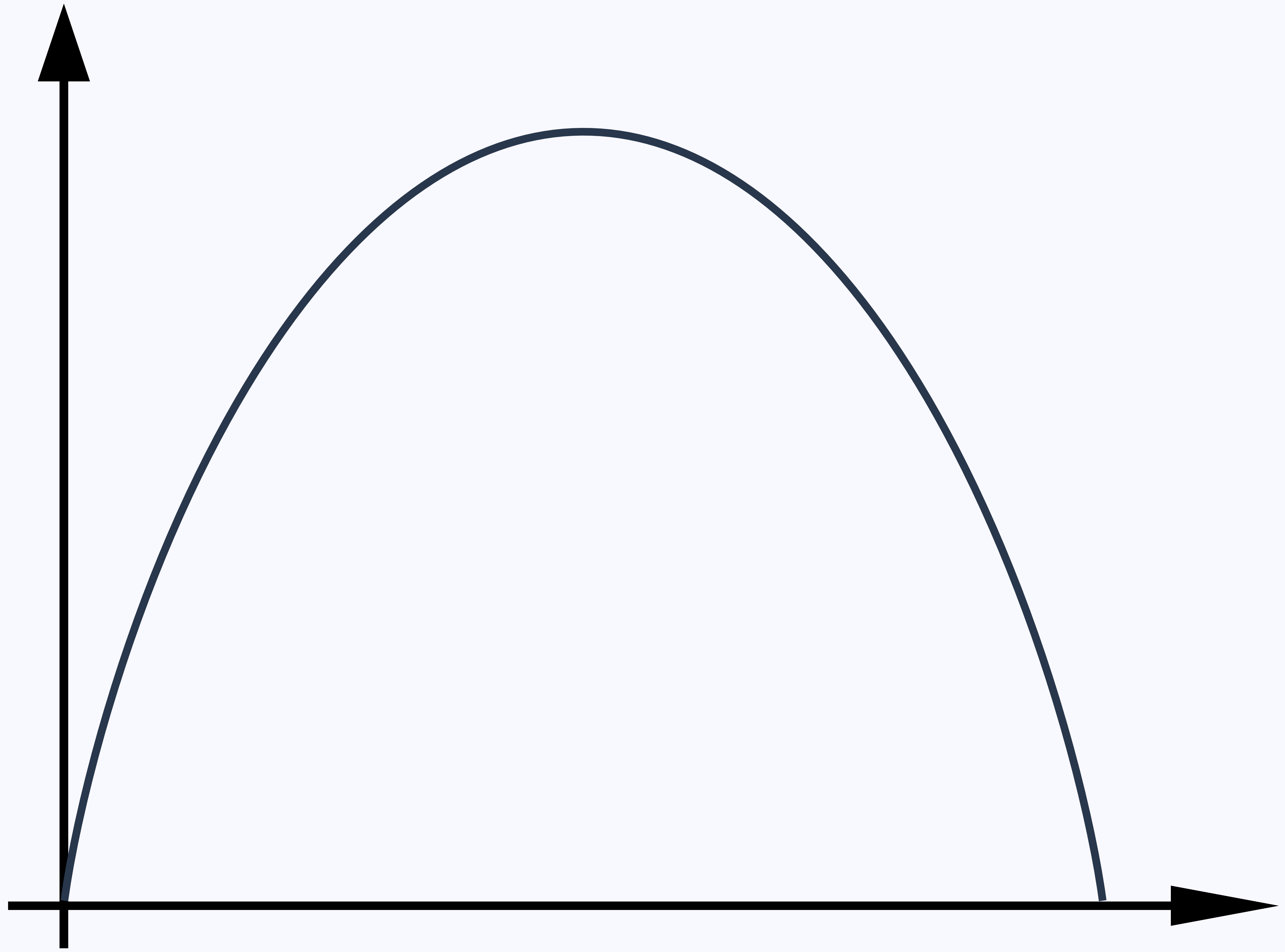


Rational Ignorance ♥ Rational Inattention

Elections with opinion polls

- $\Pr(\text{correct})$ is **independent of #voters, prior, and voting rule**
- large elections are **close regardless of a prior and voting rule**

Entropy

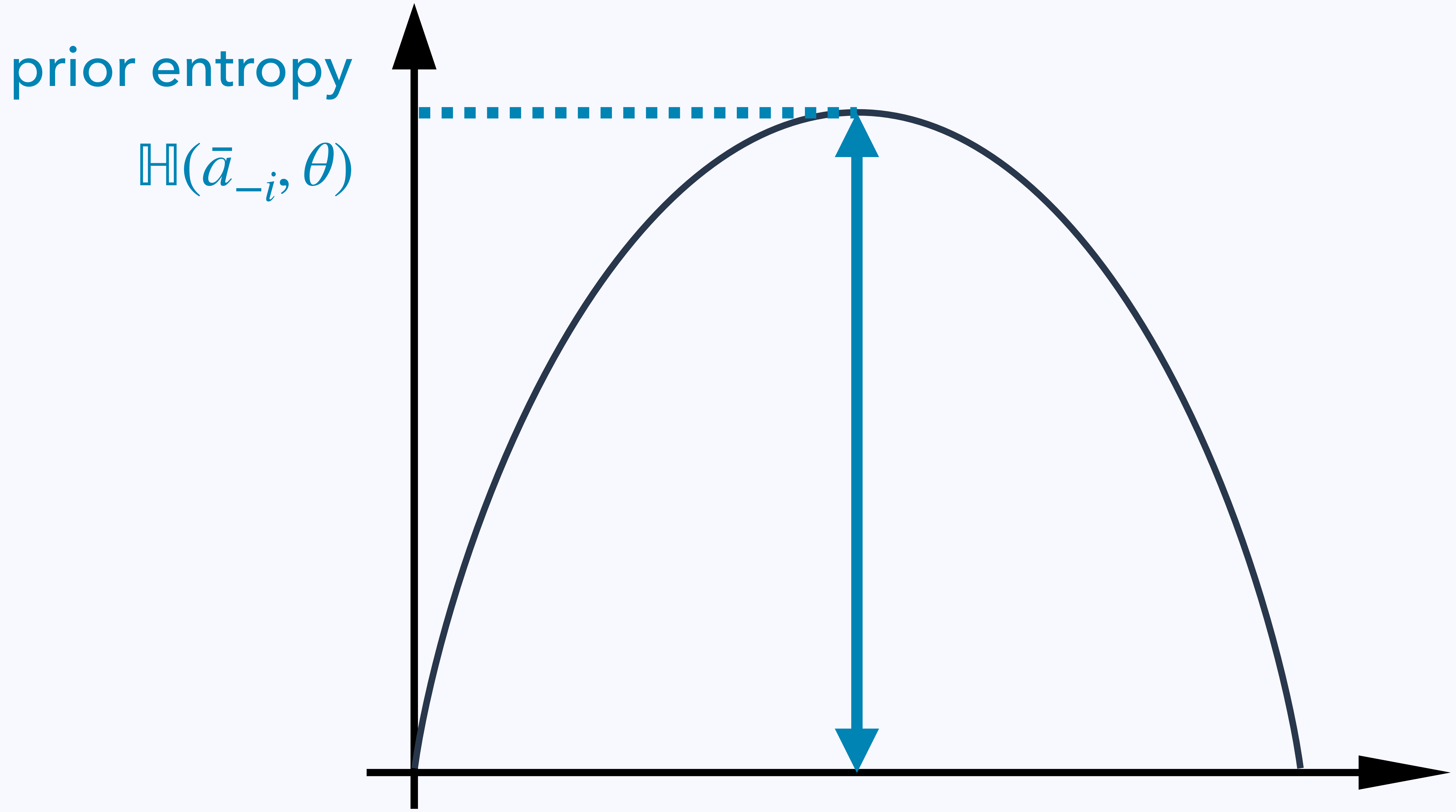


Entropy $\mathbb{H}(X)$ is a measure of uncertainty of a random variable X:

$$\mathbb{H}(X) = -\sum_x \Pr(x)\log(\Pr(x))$$

back

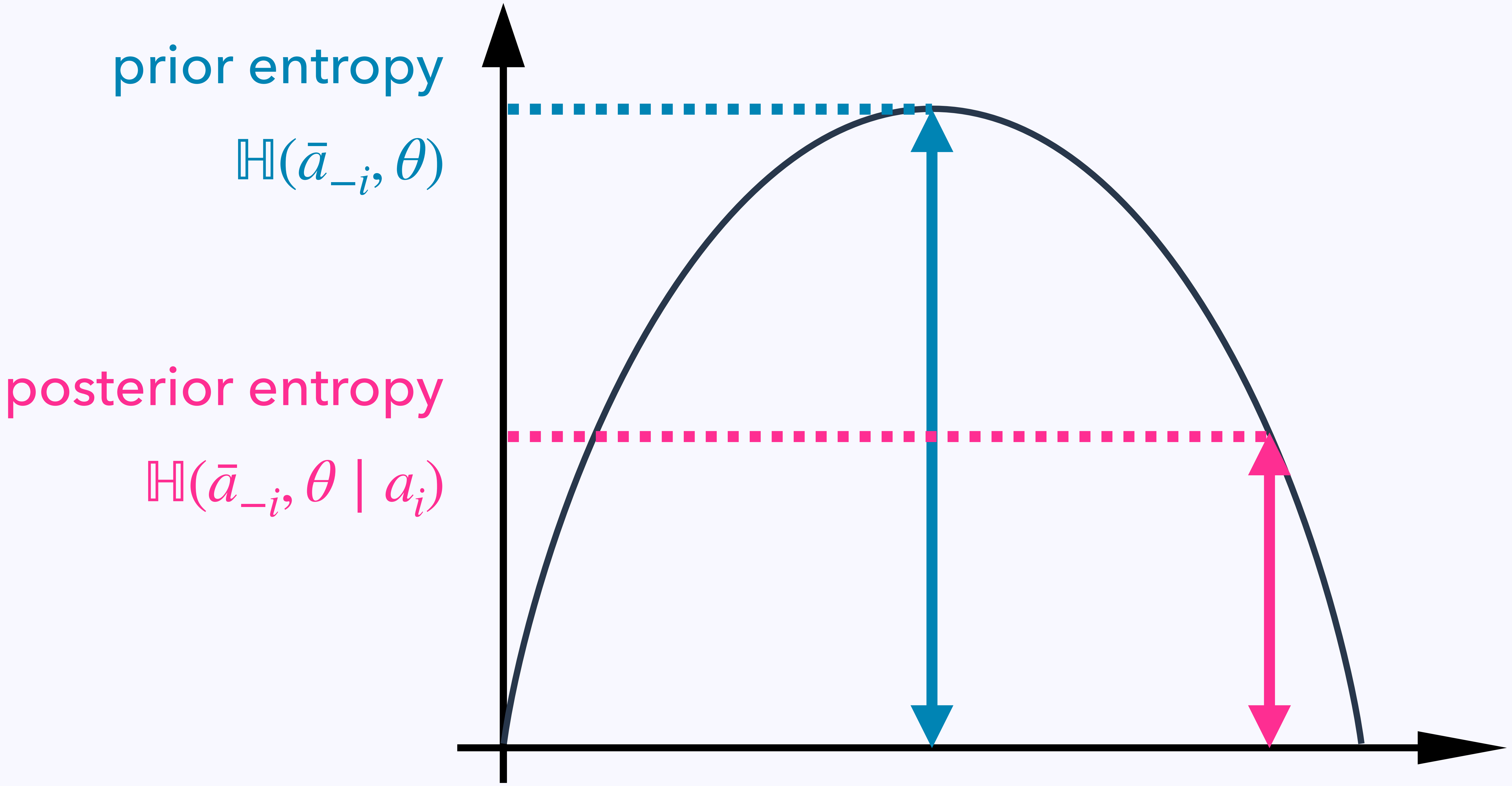
Entropy



Entropy $\mathbb{H}(X)$ is a measure of uncertainty of a random variable X:

$$\mathbb{H}(X) = -\sum_x \Pr(x) \log(\Pr(x))$$

Entropy

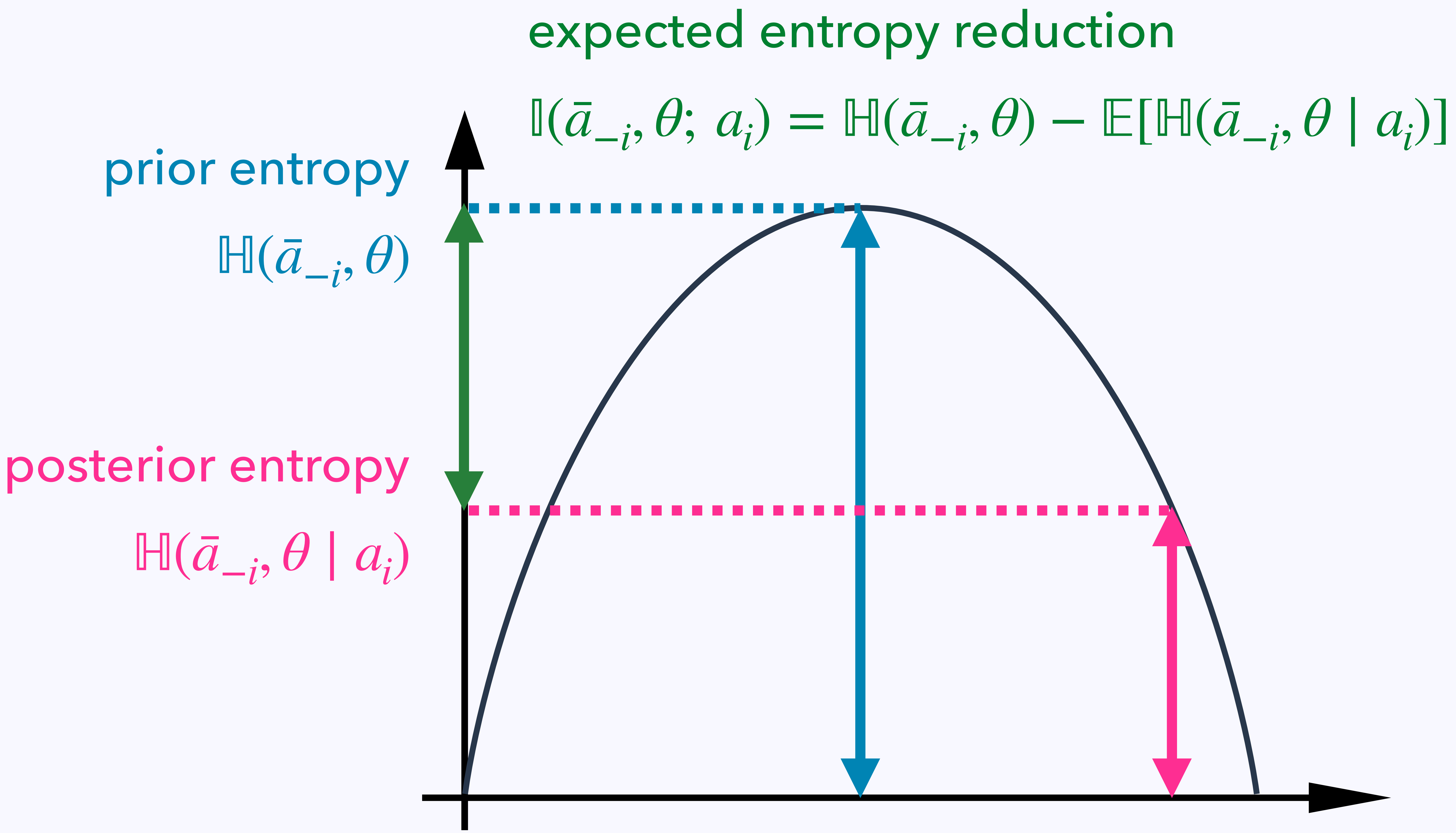


Entropy $\mathbb{H}(X)$ is a measure of uncertainty of a random variable X:

$$\mathbb{H}(X) = -\sum_x \Pr(x) \log(\Pr(x))$$

back

Entropy



Entropy $\mathbb{H}(X)$ is a measure of uncertainty of a random variable X:

$$\mathbb{H}(X) = -\sum_x \Pr(x) \log(\Pr(x))$$

back

Equilibrium Characterization

$$\Pr\left(\bar{a}_N = \frac{k}{N} \mid \theta\right) = \frac{1}{Z_N\left(p_N^*, \theta\right)} \binom{N}{k} \exp\left[\frac{u\left(\frac{k}{N}, \theta\right)}{\lambda}\right] \left(p_N^*\right)^k \left(1 - p_N^*\right)^{N-k}$$

Auxiliary game

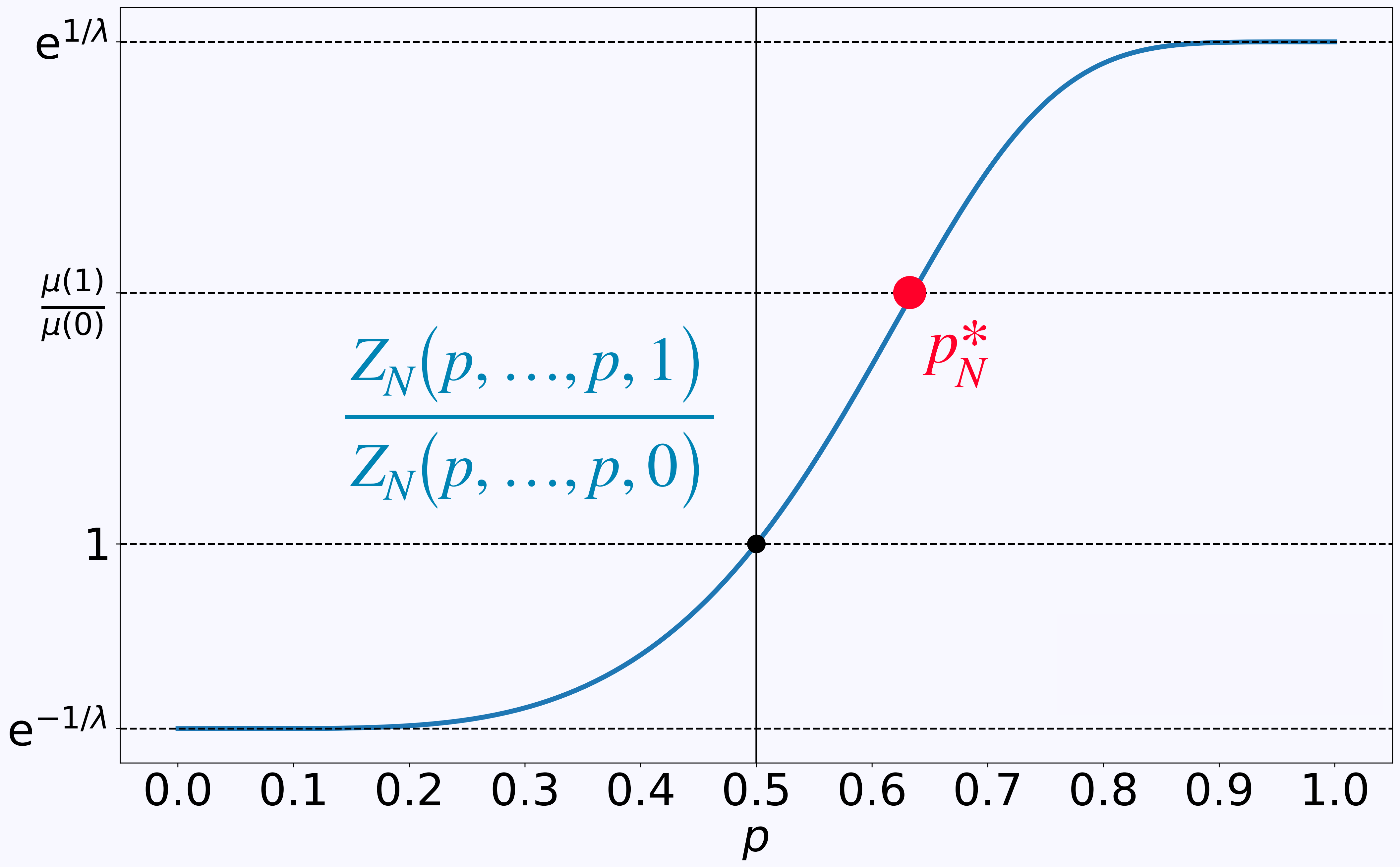
- player i chooses action $p_i \in [0,1]$ to maximize a common payoff function:

$$U_N\left(p_1, \ldots, p_N\right) = \sum_\theta \mu(\theta) \ln Z_N\left(p_1, \ldots, p_N\right)$$

$$Z_N\left(p_1, \ldots, p_N\right) = \sum_a \exp\left[\frac{u\left(\bar{a}_N, \theta\right)}{\lambda}\right] \prod_{i:a_i=1} p_i \prod_{i:a_i=0} \left(1 - p_i\right)$$

- Nash equilibrium $\left(p_N^*, \ldots, p_N^*\right)$

$$\frac{Z_N\left(p_N^*, \ldots, p_N^*, 1\right)}{Z_N\left(p_N^*, \ldots, p_N^*, 0\right)} = \frac{\mu(1)}{\mu(0)}$$



back

Proof Sketch

$$\Pr\left(\bar{a}_N > 0.5 \mid \theta = 1\right) = \sum_{k > \frac{N}{2}} \Pr\left(\bar{a}_N = \frac{k}{N} \mid \theta = 1\right)$$

$$\Pr\left(\bar{a}_N > 0.5 \mid \theta = 1\right) = \frac{1}{Z_N(p_N^*, 1)} \sum_{k > \frac{N}{2}} \binom{N}{k} \exp\left[\frac{u\left(\frac{k}{N}, 1\right)}{\lambda}\right] (p_N^*)^k (1 - p_N^*)^{N-k} = \frac{e^{1/\lambda} - \frac{\mu(0)}{\mu(1)}}{e^{1/\lambda} - e^{-1/\lambda}}$$

biased-logit

$$\Pr\left(\bar{a}_N < 0.5 \mid \theta = 0\right) = \frac{e^{1/\lambda} - \frac{\mu(1)}{\mu(0)}}{e^{1/\lambda} - e^{-1/\lambda}}$$

$$\Pr\left(\text{correct}\right) = \mu(1) \Pr\left(\bar{a}_N > 0.5 \mid \theta = 1\right) + \mu(0) \Pr\left(\bar{a}_N < 0.5 \mid \theta = 0\right) = \frac{e^{1/\lambda}}{1 + e^{1/\lambda}}$$

back

Proof Sketch

equilibrium condition:

$$\frac{Z_N(p_N^*, 1)}{Z_N(p_N^*, 0)} = \frac{\mu(1)}{\mu(0)}$$

$$\Pr\left(\bar{a}_N > 0.5 \mid \theta = 1\right) = \sum_{k > \frac{N}{2}} \Pr\left(\bar{a}_N = \frac{k}{N} \mid \theta = 1\right)$$

biased-logit

$$= \frac{1}{Z_N(p_N^*, 1)} \sum_{k > \frac{N}{2}} \binom{N}{k} \exp\left[\frac{u\left(\frac{k}{N}, 1\right)}{\lambda}\right] \left(p_N^*\right)^k \left(1 - p_N^*\right)^{N-k} = \frac{e^{1/\lambda} - \frac{\mu(0)}{\mu(1)}}{e^{1/\lambda} - e^{-1/\lambda}}$$

$$\Pr\left(\bar{a}_N < 0.5 \mid \theta = 0\right) = \frac{e^{1/\lambda} - \frac{\mu(1)}{\mu(0)}}{e^{1/\lambda} - e^{-1/\lambda}}$$

$$\Pr\left(\text{correct}\right) = \mu(1) \Pr\left(\bar{a}_N > 0.5 \mid \theta = 1\right) + \mu(0) \Pr\left(\bar{a}_N < 0.5 \mid \theta = 0\right) = \frac{e^{1/\lambda}}{1 + e^{1/\lambda}}$$

back

Proof Sketch

equilibrium condition:

$$\frac{Z_N(p_N^*, 1)}{Z_N(p_N^*, 0)} = \frac{\mu(1)}{\mu(0)}$$

$$\mathrm{Pr}\left(\bar{a}_N > 0.5 \mid \theta = 1\right) = \sum_{k > \frac{N}{2}} \mathrm{Pr}\left(\bar{a}_N = \frac{k}{N} \mid \theta = 1\right)$$

biased-logit

$$= \frac{1}{Z_N(p_N^*, 1)} \sum_{k > \frac{N}{2}} \binom{N}{k} \exp\left[\frac{u\left(\frac{k}{N}, 1\right)}{\lambda}\right] (p_N^*)^k (1 - p_N^*)^{N-k} = \frac{\mathrm{e}^{1/\lambda} - \frac{\mu(0)}{\mu(1)}}{\mathrm{e}^{1/\lambda} - \mathrm{e}^{-1/\lambda}}$$

$$\mathrm{Pr}\left(\bar{a}_N < 0.5 \mid \theta = 0\right) = \frac{\mathrm{e}^{1/\lambda} - \frac{\mu(1)}{\mu(0)}}{\mathrm{e}^{1/\lambda} - \mathrm{e}^{-1/\lambda}}$$

$$\mathrm{Pr}\left(\text{correct}\right) = \mu(1)\,\mathrm{Pr}\left(\bar{a}_N > 0.5 \mid \theta = 1\right) + \mu(0)\,\mathrm{Pr}\left(\bar{a}_N < 0.5 \mid \theta = 0\right) = \frac{\mathrm{e}^{1/\lambda}}{1 + \mathrm{e}^{1/\lambda}}$$

back

Proof Sketch

equilibrium condition:

$$\frac{Z_N(p_N^*, 1)}{Z_N(p_N^*, 0)} = \frac{\mu(1)}{\mu(0)}$$

$$\Pr\left(\bar{a}_N > 0.5 \mid \theta = 1\right) = \sum_{k > \frac{N}{2}} \Pr\left(\bar{a}_N = \frac{k}{N} \mid \theta = 1\right)$$

biased-logit

$$= \frac{1}{Z_N(p_N^*,1)} \sum_{k>\frac{N}{2}} \binom{N}{k} \exp\left[\frac{u\left(\frac{k}{N},1\right)}{\lambda}\right] (p_N^*)^k (1 - p_N^*)^{N-k} = \frac{e^{1/\lambda} - \frac{\mu(0)}{\mu(1)}}{e^{1/\lambda} - e^{-1/\lambda}}$$

$$\Pr\left(\bar{a}_N < 0.5 \mid \theta = 0\right) = \frac{e^{1/\lambda} - \frac{\mu(1)}{\mu(0)}}{e^{1/\lambda} - e^{-1/\lambda}}$$

independent of #voters N

$$\Pr\left(\text{correct}\right) = \mu(1) \Pr\left(\bar{a}_N > 0.5 \mid \theta = 1\right) + \mu(0) \Pr\left(\bar{a}_N < 0.5 \mid \theta = 0\right) = \frac{e^{1/\lambda}}{1 + e^{1/\lambda}}$$

back

Proof Sketch

equilibrium condition:

$$\frac{Z_N(p_N^*, 1)}{Z_N(p_N^*, 0)} = \frac{\mu(1)}{\mu(0)}$$

$$\text{Pr}\left(\bar{a}_N > 0.5 \mid \theta = 1\right) = \sum_{k>\frac{N}{2}} \text{Pr}\left(\bar{a}_N = \frac{k}{N} \mid \theta = 1\right)$$

biased-logit

$$= \frac{1}{Z_N(p_N^*,1)} \sum_{k>\frac{N}{2}} \binom{N}{k} \exp\left[\frac{u(\frac{k}{N},1)}{\lambda}\right] (p_N^*)^k (1 - p_N^*)^{N-k} = \frac{e^{1/\lambda} - \frac{\mu(0)}{\mu(1)}}{e^{1/\lambda} - e^{-1/\lambda}}$$

$$\text{Pr}\left(\bar{a}_N < 0.5 \mid \theta = 0\right) = \frac{e^{1/\lambda} - \frac{\mu(1)}{\mu(0)}}{e^{1/\lambda} - e^{-1/\lambda}}$$

independent of #voters N

substitute

$$\text{Pr}\left(\text{correct}\right) = \mu(1)\,\text{Pr}\left(\bar{a}_N > 0.5 \mid \theta = 1\right) + \mu(0)\,\text{Pr}\left(\bar{a}_N < 0.5 \mid \theta = 0\right) = \frac{e^{1/\lambda}}{1 + e^{1/\lambda}}$$

back

Swing Voter's Curse Feddersen–Pesendorfer 1996, 1998

Swing voter's curse when information is exogenous

- voters, when pivotal, may disregard their own signals
- being pivotal can be more informative than their own signals

No swing voter's curse in our election

- voters never acquire costly information that they will disregard in equilibrium

back

Election w/ Partisan = Supermajority-Rule Election w/o Partisan

Election with partisans are equivalent to supermajority-rule elections without partisans

$N = 101$ voters

1. $N_1 = 10$ and $N_0 = 20$ partisans for alternatives 1 and 0 respectively
   - $N - N_1 - N_0 = 71$ non-partisans
   - alternative 1 wins $\Longleftrightarrow$ 41 out of 71 vote for alternative 1
2. $N_1 = 0$ and $N_0 = 50$ partisans for alternatives 1 and 0 respectively
   - $N - N_1 - N_0 = 51$ non-partisans
   - alternative 1 wins $\Longleftrightarrow$ 41 out of 51 vote for alternative 1

back

Proof Sketch 1/2

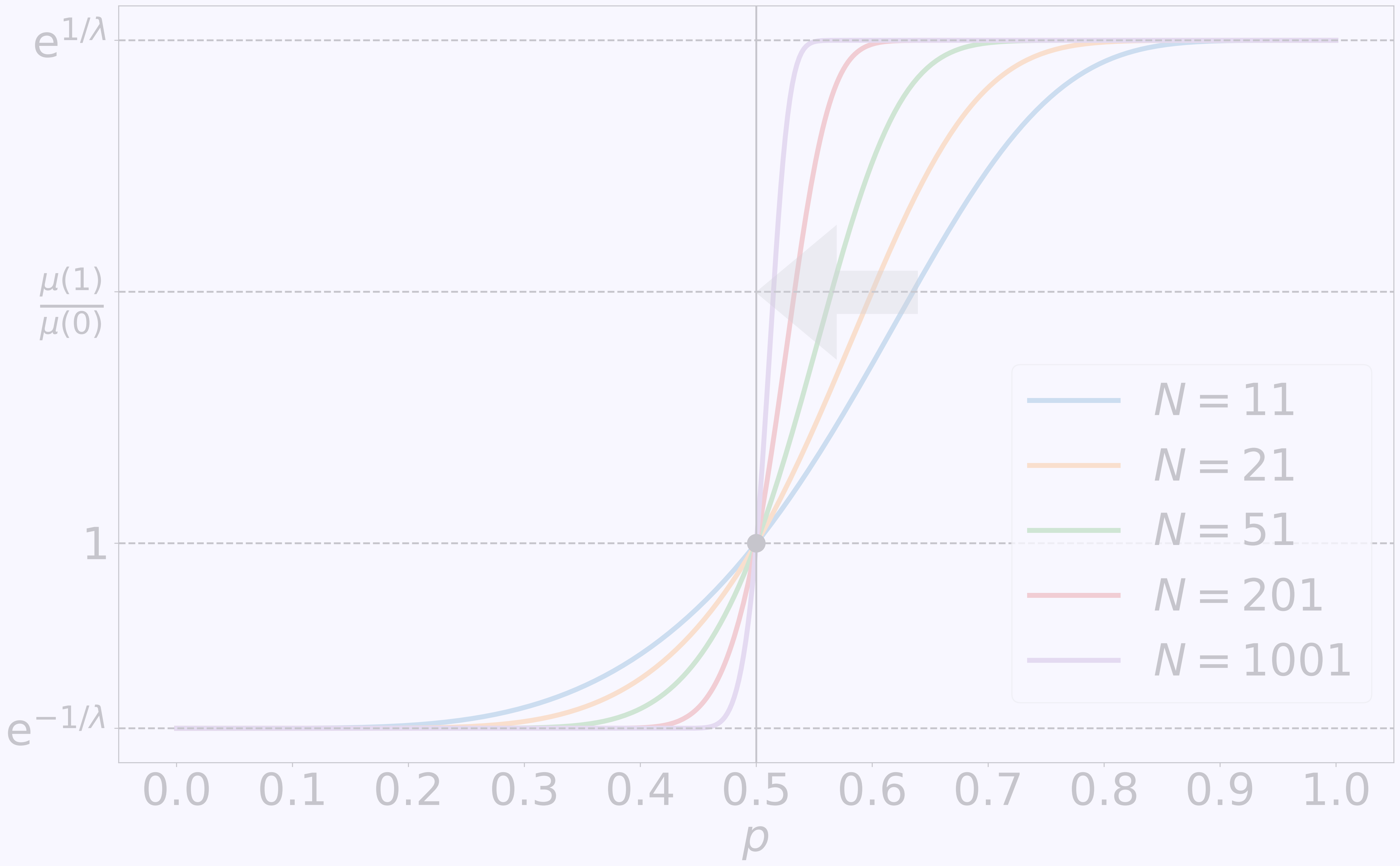
The marginal probability p_N^* of each voter choosing $a_i = 1$ converges to 0.5

p_N^* is a unique solution to

$$\frac{Z_N(p,1)}{Z_N(p,0)} = \frac{\mu(1)}{\mu(0)} \in \left(\mathrm{e}^{-1/\lambda}, \mathrm{e}^{1/\lambda}\right)$$

As $N \to \infty$

$$\frac{Z_N(p,1)}{Z_N(p,0)} \text{ becomes } \_\rceil \text{-shaped}$$



Proof Sketch 1/2

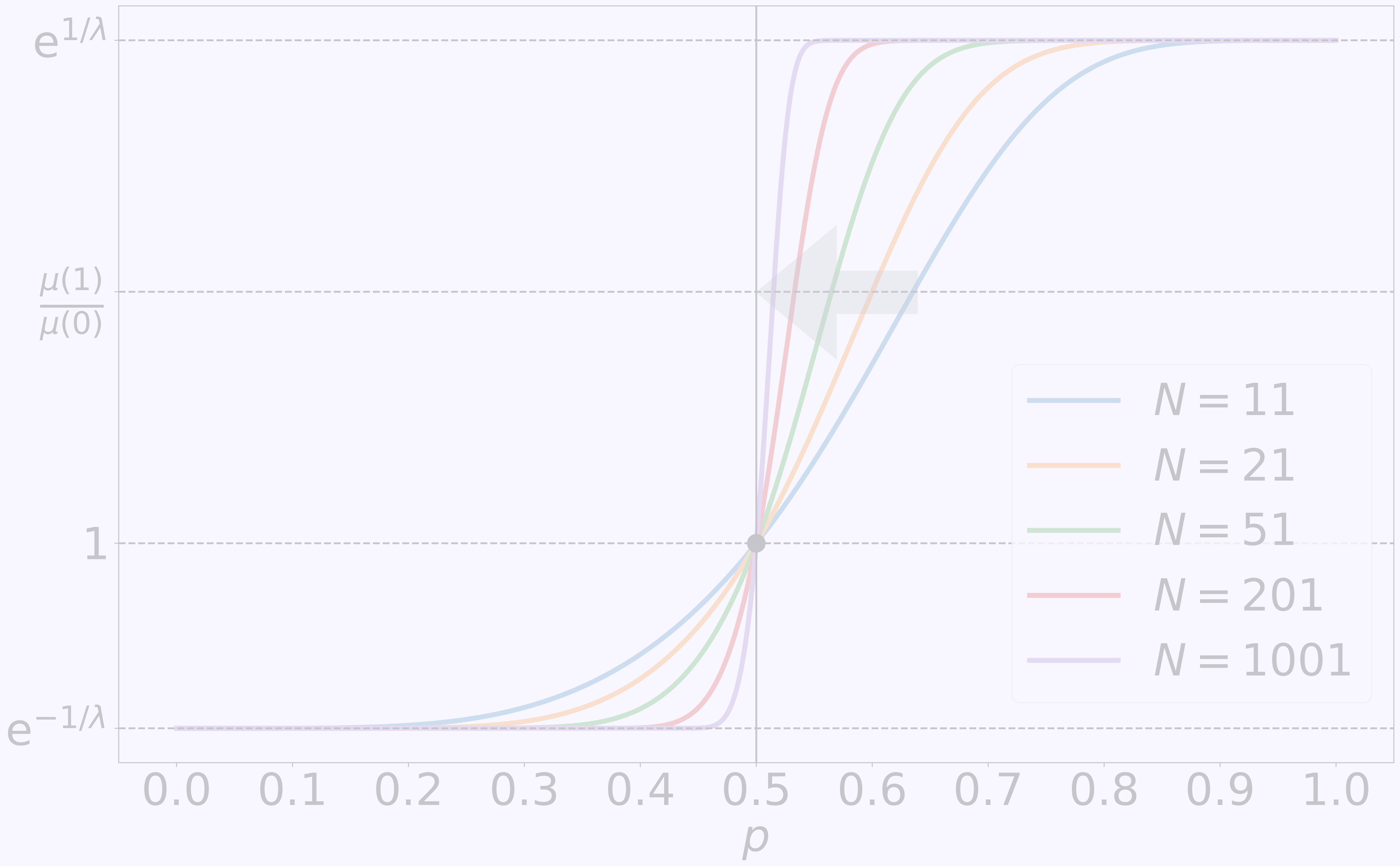
The marginal probability p_N^* of each voter choosing $a_i = 1$ converges to 0.5

p_N^* is a unique solution to

$$\frac{Z_N(p,1)}{Z_N(p,0)} = \frac{\mu(1)}{\mu(0)} \in \left(\mathrm{e}^{-1/\lambda}, \mathrm{e}^{1/\lambda}\right)$$

As $N \to \infty$

$$\frac{Z_N(p,1)}{Z_N(p,0)}$$ becomes ⌐-shaped



Proof Sketch 1/2

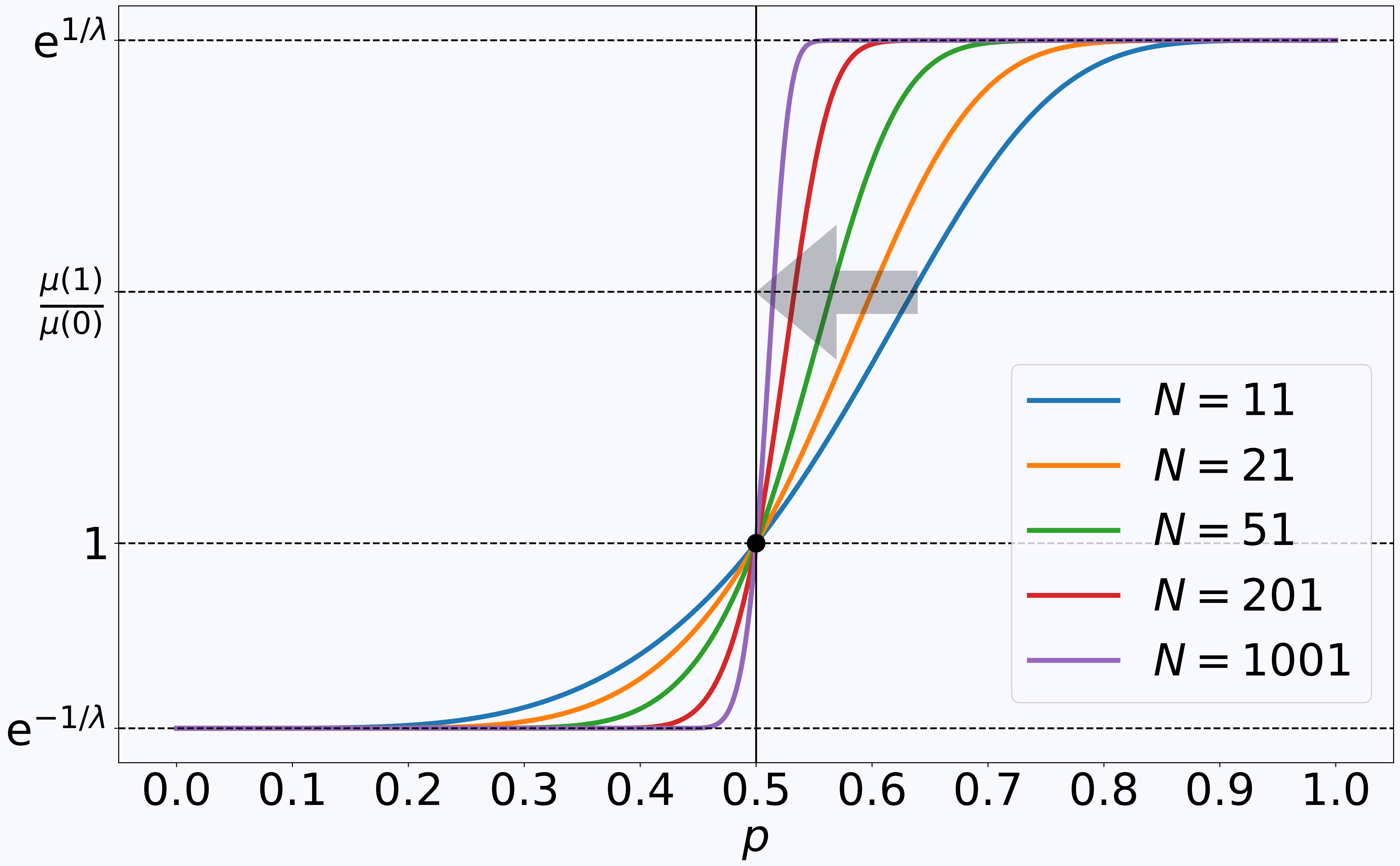
The marginal probability p_N^* of each voter choosing $a_i = 1$ converges to 0.5

p_N^* is a unique solution to

$$\frac{Z_N(p,1)}{Z_N(p,0)} = \frac{\mu(1)}{\mu(0)} \in \left(e^{-1/\lambda}, e^{1/\lambda}\right)$$

As $N \to \infty$

$$\frac{Z_N(p,1)}{Z_N(p,0)}$$ becomes ⎿⎾-shaped



Proof Sketch 2/2

biased-logit

$$\Pr\left(\left|\bar{a}_N - \frac{1}{2}\right| > \varepsilon \mid \theta\right) = \frac{\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \exp\left[\frac{u(\frac{k}{N},\theta)}{\lambda}\right] (p_N^*)^k (1-p_N^*)^{N-k}}{\sum_{k=0}^{N} \binom{N}{k} \exp\left[\frac{u(\frac{k}{N},\theta)}{\lambda}\right] (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$\leq e^{1/\lambda} \cdot \frac{\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}{\sum_{k=0}^{N} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$= e^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k} \approx e^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \frac{1}{2^N} \xrightarrow{N\to\infty} 0$$

$p_N^* \approx 1/2$

back

Proof Sketch 2/2

biased-logit

$\leq e^{1/\lambda}$ for $\|u\| \leq 1$

$$\Pr\left(\left|\bar{a}_N - \frac{1}{2}\right| > \varepsilon \mid \theta\right) = \frac{\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \boxed{\exp\left[\frac{u(\frac{k}{N},\theta)}{\lambda}\right]} (p_N^*)^k (1-p_N^*)^{N-k}}{\sum_{k=0}^{N} \binom{N}{k} \exp\left[\frac{u(\frac{k}{N},\theta)}{\lambda}\right] (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$\leq e^{1/\lambda} \cdot \frac{\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}{\sum_{k=0}^{N} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$= e^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k} \approx e^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \frac{1}{2^N} \xrightarrow{N\to\infty} 0$$

$p_N^* \approx 1/2$

back

Proof Sketch 2/2

biased-logit

$$\Pr\left(\left|\bar{a}_N - \frac{1}{2}\right| > \varepsilon \mid \theta\right) = \frac{\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \overbrace{\exp\left[\frac{u(\frac{k}{N},\theta)}{\lambda}\right]}^{\leq \mathrm{e}^{1/\lambda} \text{ for } \|u\| \leq 1} (p_N^*)^k (1-p_N^*)^{N-k}}{\sum_{k=0}^{N} \binom{N}{k} \underbrace{\exp\left[\frac{u(\frac{k}{N},\theta)}{\lambda}\right]}_{\geq 1 \text{ for } \|u\| \geq 0} (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$\leq \mathrm{e}^{1/\lambda} \cdot \frac{\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}{\sum_{k=0}^{N} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$= \mathrm{e}^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k} \approx \mathrm{e}^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \frac{1}{2^N} \xrightarrow{N\to\infty} 0$$

$p_N^* \approx 1/2$

back

Proof Sketch 2/2

biased-logit

$$\Pr\left(\left|\bar{a}_N - \frac{1}{2}\right| > \varepsilon \mid \theta\right) = \frac{\sum_{k:\left|\frac{k}{N}-\frac{1}{2}\right|>\varepsilon} \binom{N}{k} \overbrace{\exp\left[\frac{u\left(\frac{k}{N},\theta\right)}{\lambda}\right]}^{\leq e^{1/\lambda} \text{ for } \|u\| \leq 1} (p_N^*)^k (1-p_N^*)^{N-k}}{\sum_{k=0}^{N} \binom{N}{k} \underbrace{\exp\left[\frac{u\left(\frac{k}{N},\theta\right)}{\lambda}\right]}_{\geq 1 \text{ for } \|u\| \geq 0} (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$\leq e^{1/\lambda} \cdot \frac{\sum_{k:\left|\frac{k}{N}-\frac{1}{2}\right|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}{\sum_{k=0}^{N} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$= e^{1/\lambda} \cdot \sum_{k:\left|\frac{k}{N}-\frac{1}{2}\right|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k} \approx e^{1/\lambda} \cdot \sum_{k:\left|\frac{k}{N}-\frac{1}{2}\right|>\varepsilon} \binom{N}{k} \frac{1}{2^N} \xrightarrow{N\to\infty} 0$$

$p_N^* \approx 1/2$

back

Proof Sketch 2/2

biased-logit

$$\Pr\left(\left|\bar{a}_N - \frac{1}{2}\right| > \varepsilon \mid \theta\right) = \frac{\sum_{k:\left|\frac{k}{N}-\frac{1}{2}\right|>\varepsilon} \binom{N}{k} \overbrace{\exp\left[\frac{u\left(\frac{k}{N},\theta\right)}{\lambda}\right]}^{\leq e^{1/\lambda} \text{ for } \|u\| \leq 1} (p_N^*)^k (1-p_N^*)^{N-k}}{\sum_{k=0}^{N} \binom{N}{k} \underbrace{\exp\left[\frac{u\left(\frac{k}{N},\theta\right)}{\lambda}\right]}_{\geq 1 \text{ for } \|u\| \geq 0} (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$\leq e^{1/\lambda} \cdot \frac{\sum_{k:\left|\frac{k}{N}-\frac{1}{2}\right|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}{\underbrace{\sum_{k=0}^{N} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}_{= 1 \text{ by binomial theorem}}}$$

$$= e^{1/\lambda} \cdot \sum_{k:\left|\frac{k}{N}-\frac{1}{2}\right|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k} \approx e^{1/\lambda} \cdot \sum_{k:\left|\frac{k}{N}-\frac{1}{2}\right|>\varepsilon} \binom{N}{k} \frac{1}{2^N} \xrightarrow{N\to\infty} 0$$

$p_N^* \approx 1/2$

back

Proof Sketch 2/2

biased-logit

$$\Pr\left(\left|\bar{a}_N - \frac{1}{2}\right| > \varepsilon \mid \theta\right) = \frac{\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \overbrace{\exp\left[\frac{u(\frac{k}{N},\theta)}{\lambda}\right]}^{\le e^{1/\lambda} \text{ for } \|u\| \le 1} (p_N^*)^k (1-p_N^*)^{N-k}}{\sum_{k=0}^{N} \binom{N}{k} \underbrace{\exp\left[\frac{u(\frac{k}{N},\theta)}{\lambda}\right]}_{\ge 1 \text{ for } \|u\| \ge 0} (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$\le e^{1/\lambda} \cdot \frac{\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}{\underbrace{\sum_{k=0}^{N} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}_{= 1 \text{ by binomial theorem}}}$$

$$= e^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k} \approx e^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \frac{1}{2^N} \xrightarrow{N\to\infty} 0$$

$p_N^* \approx 1/2$

back

Proof Sketch 2/2

biased-logit

$$\Pr\left(\left|\bar{a}_N - \frac{1}{2}\right| > \varepsilon \mid \theta\right) = \frac{\displaystyle\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \overbrace{\exp\left[\frac{u(\frac{k}{N},\theta)}{\lambda}\right]}^{\leq e^{1/\lambda} \text{ for } \|u\| \leq 1} (p_N^*)^k (1-p_N^*)^{N-k}}{\displaystyle\sum_{k=0}^{N} \binom{N}{k} \underbrace{\exp\left[\frac{u(\frac{k}{N},\theta)}{\lambda}\right]}_{\geq 1 \text{ for } \|u\| \geq 0} (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$\leq e^{1/\lambda} \cdot \frac{\displaystyle\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}{\underbrace{\displaystyle\sum_{k=0}^{N} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}_{= 1 \text{ by binomial theorem}}}$$

$$= e^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k} \approx e^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \frac{1}{2^N} \xrightarrow{N\to\infty} 0$$

$p_N^* \approx 1/2$

back

Proof Sketch 2/2

biased-logit

$$\Pr\left(\left|\bar{a}_N - \frac{1}{2}\right| > \varepsilon \mid \theta\right) = \frac{\displaystyle\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \overbrace{\exp\left[\frac{u\left(\frac{k}{N},\theta\right)}{\lambda}\right]}^{\leq e^{1/\lambda} \text{ for } \|u\| \leq 1} (p_N^*)^k (1-p_N^*)^{N-k}}{\displaystyle\sum_{k=0}^{N} \binom{N}{k} \underbrace{\exp\left[\frac{u\left(\frac{k}{N},\theta\right)}{\lambda}\right]}_{\geq 1 \text{ for } \|u\| \geq 0} (p_N^*)^k (1-p_N^*)^{N-k}}$$

$$\leq e^{1/\lambda} \cdot \frac{\displaystyle\sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}{\underbrace{\displaystyle\sum_{k=0}^{N} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k}}_{= 1 \text{ by binomial theorem}}}$$

$$= e^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} (p_N^*)^k (1-p_N^*)^{N-k} \approx e^{1/\lambda} \cdot \sum_{k:|\frac{k}{N}-\frac{1}{2}|>\varepsilon} \binom{N}{k} \frac{1}{2^N} \xrightarrow{N\to\infty} 0$$

$p_N^* \approx 1/2$

back

Election w/ Partisan = Supermajority-Rule Election w/o Partisan

Election with partisans are equivalent to supermajority-rule elections without partisans

$N = 101$ voters

1. $N_1 = 10$ and $N_0 = 20$ partisans for alternatives 1 and 0 respectively
   - $N - N_1 - N_0 = 71$ non-partisans
   - alternative 1 wins $\Longleftrightarrow$ 41 out of 71 vote for alternative 1
2. $N_1 = 0$ and $N_0 = 50$ partisans for alternatives 1 and 0 respectively
   - $N - N_1 - N_0 = 51$ non-partisans
   - alternative 1 wins $\Longleftrightarrow$ 41 out of 51 vote for alternative 1

back

Proof Sketch 1/3

Given the symmetric prior $\mu(1) = \mu(0) = 1/2$,

$$\Pr\left(u(\bar{b}_N, \theta) = 1\right) \xrightarrow{N\to\infty} \Phi(2x) \quad \text{with} \quad 2\lambda x = \phi(2x)$$

Given state $\theta = 1$, voters choose actions $b_i = 1$ and $b_i = 0$ with probabilities $\frac{1}{2} + t_N$ and $\frac{1}{2} - t_N$

- $b_i \,|_{\theta=1} \sim$ Bernoulli$\left(\frac{1}{2} + t_N\right)$ with mean μ_N and variance $(\sigma_N)^2$

$$\Pr\left(\bar{b}_N > \frac{1}{2} \mid \theta = 1\right) = \Pr\left(\frac{\sqrt{N}(\bar{b}_N - \mu_N)}{\sigma_N} > -\frac{\sqrt{N}t_N}{\sigma_N} \mid \theta = 1\right) \to \lim_{N\to\infty} \Phi\left(\frac{\sqrt{N}t_N}{\sigma_N}\right) = \Phi\left(2 \lim_{N\to\infty} \sqrt{N}t_N\right)$$

Berry-Esseen theorem

$$\Pr\left(u(\bar{b}_N, \theta) = 1\right) = \mu(1)\Pr\left(\bar{b}_N > \frac{1}{2} \mid \theta = 1\right) + \mu(0)\Pr\left(\bar{b}_N < \frac{1}{2} \mid \theta = 0\right) \xrightarrow{N\to\infty} \Phi\left(2 \lim_{N\to\infty} \sqrt{N}t_N\right)$$

Proof Sketch 1/3

Given the symmetric prior $\mu(1) = \mu(0) = 1/2$,

$$\Pr\left(u\left(\bar{b}_N, \theta\right) = 1\right) \xrightarrow{N\to\infty} \Phi(2x) \quad \text{with} \quad 2\lambda x = \phi(2x)$$

Given state $\theta = 1$, voters choose actions $b_i = 1$ and $b_i = 0$ with probabilities $\frac{1}{2} + t_N$ and $\frac{1}{2} - t_N$

- $b_i \,|_{\theta=1} \sim \text{Bernoulli}\left(\frac{1}{2} + t_N\right)$ with mean μ_N and variance $(\sigma_N)^2$

$$\Pr\left(\bar{b}_N > \frac{1}{2} \mid \theta = 1\right) = \Pr\left(\frac{\sqrt{N}(\bar{b}_N - \mu_N)}{\sigma_N} > -\frac{\sqrt{N}t_N}{\sigma_N} \mid \theta = 1\right) \to \lim_{N\to\infty} \Phi\left(\frac{\sqrt{N}t_N}{\sigma_N}\right) = \Phi\left(2 \lim_{N\to\infty} \sqrt{N}t_N\right)$$

Berry-Esseen theorem

$$\Pr\left(u\left(\bar{b}_N, \theta\right) = 1\right) = \mu(1)\Pr\left(\bar{b}_N > \frac{1}{2} \mid \theta = 1\right) + \mu(0)\Pr\left(\bar{b}_N < \frac{1}{2} \mid \theta = 0\right) \xrightarrow{N\to\infty} \Phi\left(2 \lim_{N\to\infty} \sqrt{N}t_N\right)$$

Given the symmetric prior $\mu(1) = \mu(0) = 1/2$,

$$\Pr\big(u(\bar{b}_N, \theta) = 1\big) \xrightarrow{N\to\infty} \Phi(2x) \quad \text{with} \quad 2\lambda x = \phi(2x)$$

Given state $\theta = 1$, voters choose actions $b_i = 1$ and $b_i = 0$ with probabilities $\frac{1}{2} + t_N$ and $\frac{1}{2} - t_N$

- $b_i \,|_{\theta=1} \sim \text{Bernoulli}\big(\frac{1}{2} + t_N\big)$ with mean μ_N and variance $(\sigma_N)^2$

$$\Pr\left(\bar{b}_N > \frac{1}{2} \mid \theta = 1\right) = \Pr\left(\frac{\sqrt{N}(\bar{b}_N - \mu_N)}{\sigma_N} > -\frac{\sqrt{N}t_N}{\sigma_N} \mid \theta = 1\right) \to \lim_{N\to\infty} \Phi\left(\frac{\sqrt{N}t_N}{\sigma_N}\right) = \Phi\left(2 \lim_{N\to\infty} \sqrt{N}t_N\right)$$

Berry-Esseen theorem

$$\Pr\big(u(\bar{b}_N, \theta) = 1\big) = \mu(1)\Pr\left(\bar{b}_N > \frac{1}{2} \mid \theta = 1\right) + \mu(0)\Pr\left(\bar{b}_N < \frac{1}{2} \mid \theta = 0\right) \xrightarrow{N\to\infty} \Phi\left(2 \lim_{N\to\infty} \sqrt{N}t_N\right)$$

Proof Sketch 1/3

Given the symmetric prior $\mu(1) = \mu(0) = 1/2$,

$$\Pr\big(u(\bar{b}_N, \theta) = 1\big) \xrightarrow{N\to\infty} \Phi(2x) \quad \text{with} \quad 2\lambda x = \phi(2x)$$

Given state $\theta = 1$, voters choose actions $b_i = 1$ and $b_i = 0$ with probabilities $\frac{1}{2} + t_N$ and $\frac{1}{2} - t_N$

- $b_i \,|_{\theta=1} \sim \text{Bernoulli}\big(\frac{1}{2} + t_N\big)$ with mean μ_N and variance $(\sigma_N)^2$

$$\Pr\left(\bar{b}_N > \frac{1}{2} \mid \theta = 1\right) = \Pr\left(\frac{\sqrt{N}(\bar{b}_N - \mu_N)}{\sigma_N} > -\frac{\sqrt{N}t_N}{\sigma_N} \mid \theta = 1\right) \to \lim_{N\to\infty} \Phi\left(\frac{\sqrt{N}t_N}{\sigma_N}\right) = \Phi\left(2 \lim_{N\to\infty} \sqrt{N}t_N\right)$$

Berry-Esseen theorem

$$\Pr\big(u(\bar{b}_N, \theta) = 1\big) = \mu(1)\Pr\left(\bar{b}_N > \frac{1}{2} \mid \theta = 1\right) + \mu(0)\Pr\left(\bar{b}_N < \frac{1}{2} \mid \theta = 0\right) \xrightarrow{N\to\infty} \Phi\left(2 \lim_{N\to\infty} \sqrt{N}t_N\right)$$

Proof Sketch 2/3

The symmetric equilibrium strategy $Q_N^*(b_i \mid \theta)$ is optimal: $\max_{Q_i} \; \mathbb{E}\left[u(\bar{a}_N, \theta)\right]$ − unit cost $\lambda \cdot \mathbb{I}(\theta;\, b_i)$

First-order condition:

$$\binom{2n}{n}\left(\frac{1}{2}+t_N\right)^n\left(\frac{1}{2}-t_N\right)^n = \lambda \log\frac{\frac{1}{2}+t_N}{\frac{1}{2}-t_N}$$

#voters $N = 2n+1$

Stirling's formula on LHS and Taylor-expansion on RHS:

$$\binom{2n}{n}\frac{\sqrt{n}}{2^{2n}} \to \frac{1}{\sqrt{\pi}}$$

$$\frac{1}{\sqrt{\pi n}}\left(1-4t_N^2\right)^n \approx \lambda\left(\frac{\mathrm{d}}{\mathrm{d}t_N}\log\frac{\frac{1}{2}+t_N}{\frac{1}{2}-t_N}\bigg|_{t_N=0}\right)t_N$$

Rearrange the terms

$$\frac{1}{\sqrt{2\pi}}\exp\left(-\left(2\sqrt{N}\,t_N\right)^2\right) \approx \lambda \cdot 2\sqrt{N}\,t_N$$

Proof Sketch 2/3

The symmetric equilibrium strategy $Q_N^*(b_i \mid \theta)$ is optimal: $\max_{Q_i} \; \mathbb{E}\left[u(\bar{a}_N, \theta)\right]$ − unit cost $\lambda \cdot \mathbb{I}(\theta;\, b_i)$

First-order condition:

$$\underbrace{\binom{2n}{n}\left(\frac{1}{2}+t_N\right)^n\left(\frac{1}{2}-t_N\right)^n}_{\text{probability of being pivotal}} = \underbrace{\lambda \log \frac{\frac{1}{2}+t_N}{\frac{1}{2}-t_N}}_{\text{marginal cost}}$$

#voters $N = 2n + 1$

Stirling's formula on LHS and Taylor-expansion on RHS:

$$\binom{2n}{n}\frac{\sqrt{n}}{2^{2n}} \to \frac{1}{\sqrt{\pi}}$$

$$\frac{1}{\sqrt{\pi n}}\left(1-4t_N^2\right)^n \approx \lambda \left(\frac{\mathrm{d}}{\mathrm{d}t_N} \log \frac{\frac{1}{2}+t_N}{\frac{1}{2}-t_N}\Bigg|_{t_N=0}\right) t_N$$

Rearrange the terms

$$\frac{1}{\sqrt{2\pi}} \exp\left(-\left(2\sqrt{N}\,t_N\right)^2\right) \approx \lambda \cdot 2\sqrt{N}\,t_N$$

Proof Sketch 2/3

The symmetric equilibrium strategy $Q_N^*(b_i \mid \theta)$ is optimal: $\max_{Q_i} \; \mathbb{E}\left[u(\bar{a}_N, \theta)\right]$ − unit cost $\lambda \cdot \mathbb{I}(\theta;\, b_i)$

First-order condition:

$$\underbrace{\binom{2n}{n}\left(\frac{1}{2}+t_N\right)^n\left(\frac{1}{2}-t_N\right)^n}_{\text{probability of being pivotal}} = \underbrace{\lambda \log \frac{\frac{1}{2}+t_N}{\frac{1}{2}-t_N}}_{\text{marginal cost}}$$

#voters $N = 2n + 1$

Stirling's formula on LHS and Taylor-expansion on RHS:

$$\binom{2n}{n}\frac{\sqrt{n}}{2^{2n}} \to \frac{1}{\sqrt{\pi}}$$

$$\frac{1}{\sqrt{\pi n}}\left(1-4t_N^2\right)^n \approx \lambda \left(\frac{\mathrm{d}}{\mathrm{d}t_N}\log\frac{\frac{1}{2}+t_N}{\frac{1}{2}-t_N}\bigg|_{t_N=0}\right)t_N$$

Rearrange the terms

$$\frac{1}{\sqrt{2\pi}}\exp\left(-\left(2\sqrt{N}\,t_N\right)^2\right) \approx \lambda \cdot 2\sqrt{N}\,t_N$$

Proof Sketch 2/3

The symmetric equilibrium strategy $Q_N^*(b_i \mid \theta)$ is optimal: $\max_{Q_i} \; \mathbb{E}\left[u(\bar{a}_N, \theta)\right]$ − unit cost $\lambda \cdot \mathbb{I}(\theta;\, b_i)$

First-order condition:

$$\underbrace{\binom{2n}{n}\left(\frac{1}{2}+t_N\right)^n\left(\frac{1}{2}-t_N\right)^n}_{\text{probability of being pivotal}} = \underbrace{\lambda \log\frac{\frac{1}{2}+t_N}{\frac{1}{2}-t_N}}_{\text{marginal cost}}$$

#voters $N = 2n+1$

Stirling's formula on LHS and Taylor-expansion on RHS:

$$\binom{2n}{n}\frac{\sqrt{n}}{2^{2n}} \to \frac{1}{\sqrt{\pi}}$$

$$\frac{1}{\sqrt{\pi n}}\left(1-4t_N^2\right)^n \approx \lambda \left(\underbrace{\frac{\mathrm{d}}{\mathrm{d}t_N}\log\frac{\frac{1}{2}+t_N}{\frac{1}{2}-t_N}\bigg|_{t_N=0}}_{=4}\right) t_N$$

$= 4\, t_N$

Rearrange the terms

$$\frac{1}{\sqrt{2\pi}}\exp\left(-\left(2\sqrt{N}\,t_N\right)^2\right) \approx \lambda \cdot 2\sqrt{N}\,t_N$$

Proof Sketch 2/3

The symmetric equilibrium strategy $Q_N^*(b_i \mid \theta)$ is optimal: $\max_{Q_i} \; \mathbb{E}\left[u(\bar{a}_N, \theta)\right]$ − unit cost $\lambda \cdot \mathbb{I}(\theta;\, b_i)$

First-order condition:

$$\underbrace{\binom{2n}{n}\left(\frac{1}{2}+t_N\right)^n\left(\frac{1}{2}-t_N\right)^n}_{\text{probability of being pivotal}} = \underbrace{\lambda \log\frac{\frac{1}{2}+t_N}{\frac{1}{2}-t_N}}_{\text{marginal cost}}$$

#voters $N = 2n+1$

Stirling's formula on LHS and Taylor-expansion on RHS:

$$\binom{2n}{n}\frac{\sqrt{n}}{2^{2n}} \to \frac{1}{\sqrt{\pi}}$$

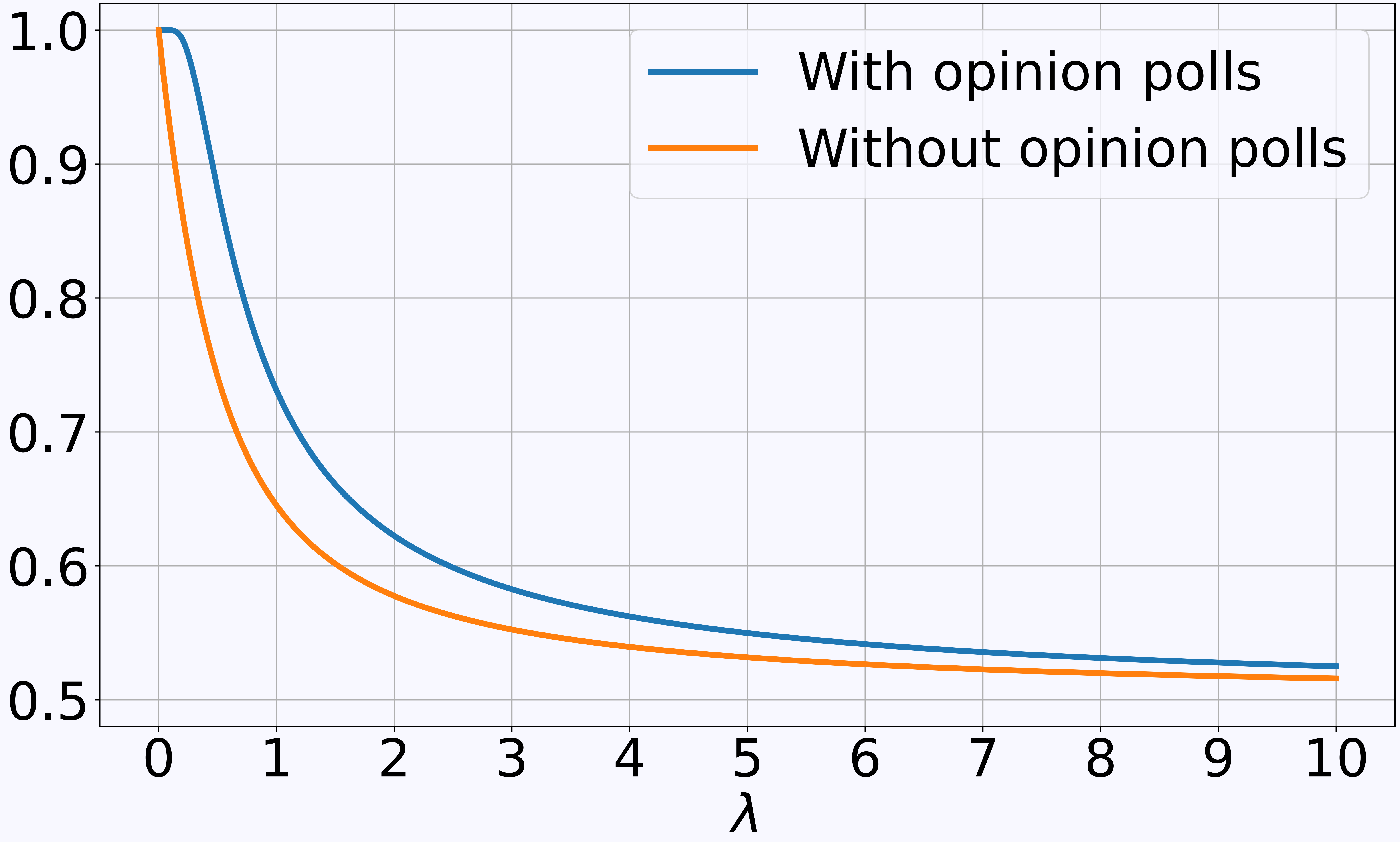
$$\frac{1}{\sqrt{\pi n}}\left(1-4t_N^2\right)^n \approx \lambda \underbrace{\left(\frac{\mathrm{d}}{\mathrm{d}t_N}\log\frac{\frac{1}{2}+t_N}{\frac{1}{2}-t_N}\Bigg|_{t_N=0}\right)t_N}_{=4\,t_N}$$

Rearrange the terms

$$\underbrace{\frac{1}{\sqrt{2\pi}}\exp\left(-\left(2\sqrt{N}\,t_N\right)^2\right)}_{=\phi\left(2\sqrt{N}\,t_N\right)} \approx \lambda \cdot 2\sqrt{N}\,t_N$$

Proof Sketch 3/3

$$\frac{\mathrm{e}^{1/\lambda}}{1+\mathrm{e}^{1/\lambda}} = \Pr\big(u\big(\bar{a}_N,\theta\big) = 1\big) > \lim_{N\to\infty} \Pr\big(u\big(\bar{b}_N,\theta\big) = 1\big) = \Phi(2x) \quad \text{with} \quad 2\lambda x = \phi(2x)$$



back

Berry–Esseen Theorem

Let $X_1, \ldots, X_N$ be iid with mean $\mu = 0$ and variance $\sigma^2 > 0$

- sample average $\bar{X}_N = \frac{1}{N} \sum_i X_i$

The **Berry–Esseen theorem** is a quantitative version of the central limit theorem:

$$\left| \Pr\left(\frac{\sqrt{N}\bar{X}_N}{\sigma} \leq x \right) - \Phi(x) \right| \leq \frac{3\mathbb{E}\,|X|^3}{\sigma^3 \sqrt{N}}$$

The central limit theorem

$$\Pr\left(\frac{\sqrt{N}\bar{X}_N}{\sigma} \leq x \right) \to \Phi(x)$$

Why Not Continuum-Voter Model?

Continuum-voter models

- Pr(pivotal) $= 0 \implies$ voters acquire **no** information

  $\implies \nexists$ informative equilibrium
- probability of correct choice $= \mu(1), \mu(0)$

Finite-voter models

- Pr(pivotal) $> 0 \implies$ voters acquire some information

  $\implies \exists$ informative equilibrium
- probability of correct choice $= \mathrm{e}^{1/\lambda} / \left(1 + \mathrm{e}^{1/\lambda}\right) > \mu(1), \mu(0)$

back

RDD in Close Elections

RDD is an empirical strategy for identifying treatment effects of election outcomes

- close elections are a quasi-natural experiment
- bare winners and bare losers **should be similar** (Hahn–Todd–van der Klaauw 2001)

Examples

- incumbency advantage in future elections (Lee 2008)
- post-electoral legislative votes (Lee–Moretti–Butler 2004)
- drug trafficking control in Mexico (Dell 2015)
- federal budget process in Brazil (Firpo–Ponczek–Sanfelice 2015)

RDD in Close Elections

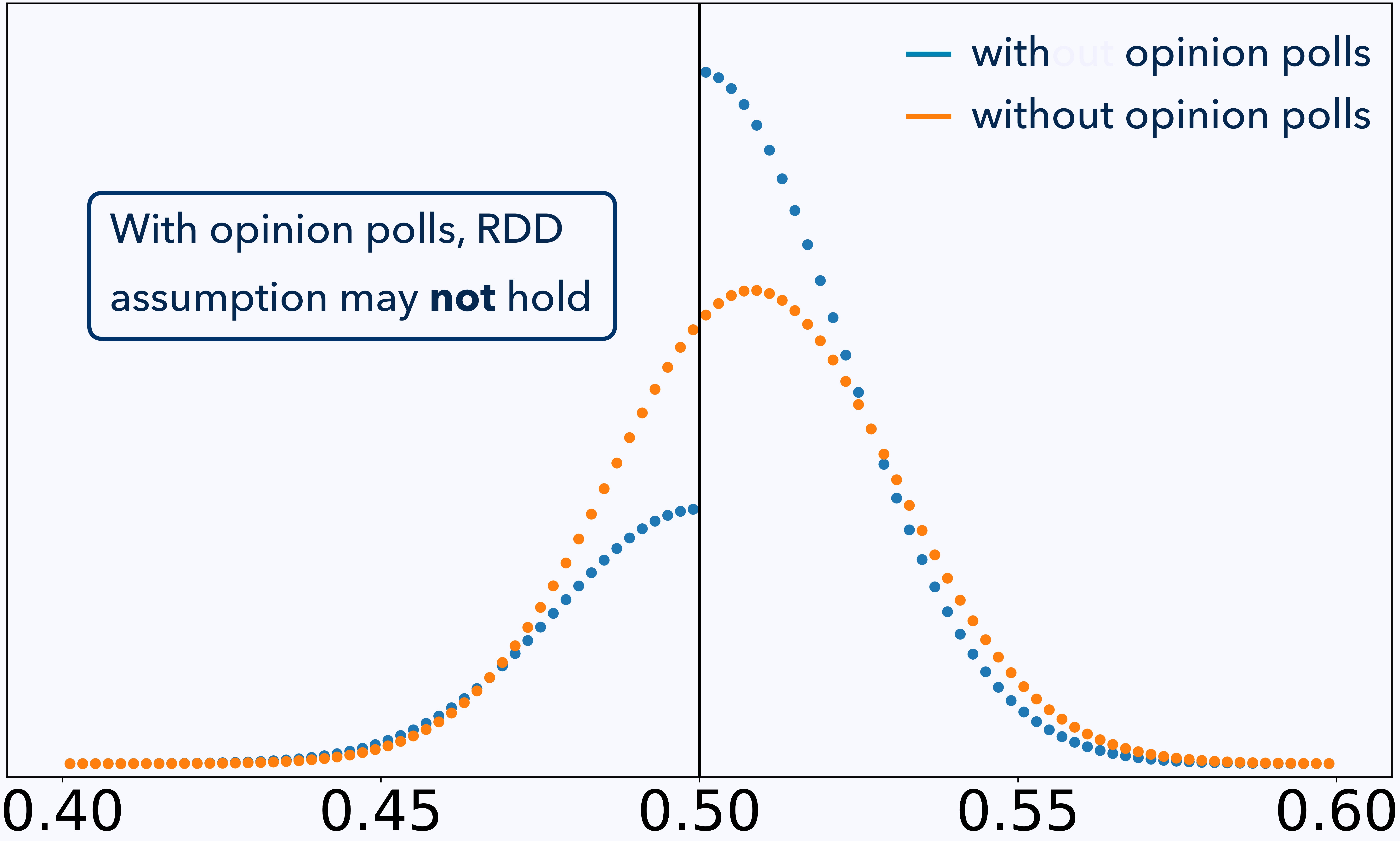
Mixed evidence on RDD validity in close elections

- winners and losers differ in close US House elections (Caughey–Sekhon 2011)
- incumbents systematically win close US House elections (Snyder 2005)
- but not the case in many other close elections (Eggers et al. 2015)

What might violate the RDD assumption — especially, in close US House elections

- U.S. House elections are **frequently polled** (Eggers et al. 2015)

RDD Assumption



x-axis: vote share $\bar{a}_N = x$ y-axis: $\Pr(\bar{a}_N = x \mid \theta = 1)$

back